3 Epidemic Data

McMaster University

Mathematics and Statistics

$$\int_M d\omega = \int_{\partial M} \omega$$

Mathematics 4MB3/6MB3 Mathematical Biology

Instructor: David Earn

Lecture 3
Epidemic Data
Monday 23 September 2019

Announcements

- You should have received an invitation to do the contributions survey for Assignment 1. Please do it TODAY (*e.g.*, during the mid-class break).
- Don't stress about the ratings about each other's contributions. The issue is whether some group members did not pull their weight. If somebody didn't try and others had to pick up the slack, that person should be penalized. I will not penalize somebody because they tried but felt they didn't contribute as much to the final document as they could have. Do try to even out the work across the assignments.
- Make sure everyone in your group gets a chance to be in control of the LaTeX for one assignment.

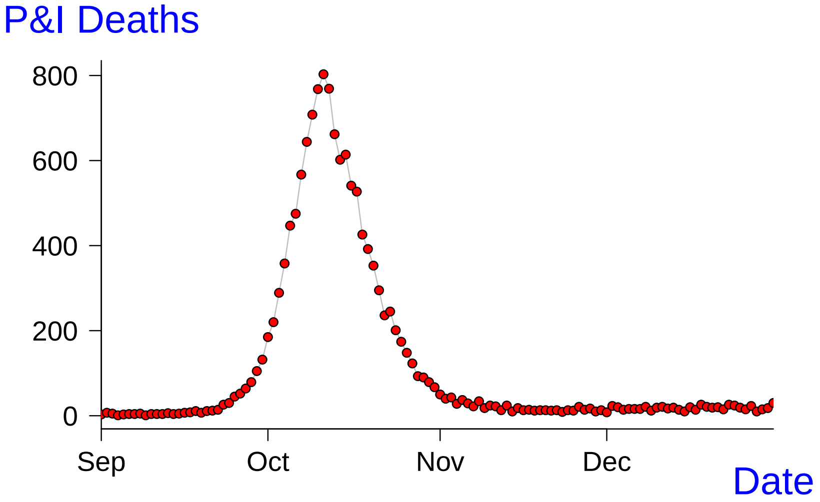
More Announcements!

- **Assignment 2:**
  Due Monday 7 October 2019 by e-mail before class.
- **Midterm test:**
  - *Date:* Monday 4 November 2019
  - *Time:* 11:30am–1:30pm
  - *Location:* in class, ETB-237

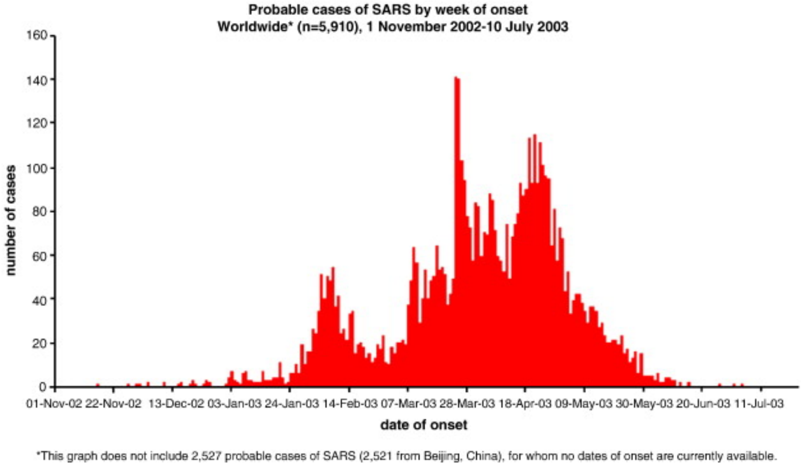
Attendance

Who is here?

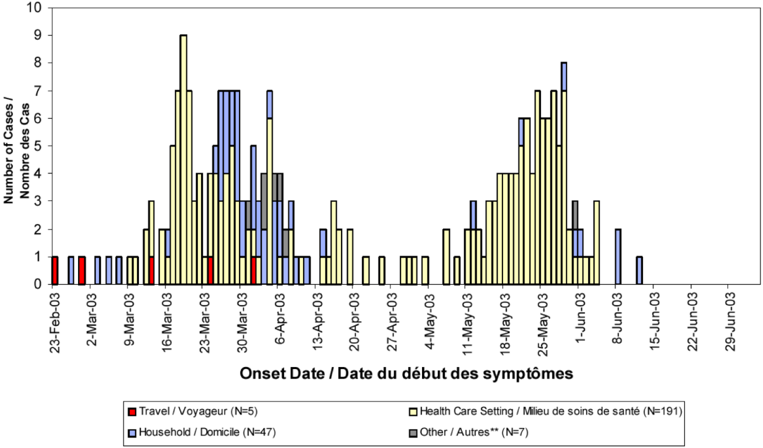
P&I Mortality, Philadelphia, 1918



SARS in 2003 (Worldwide)



SARS in 2003 (Toronto)

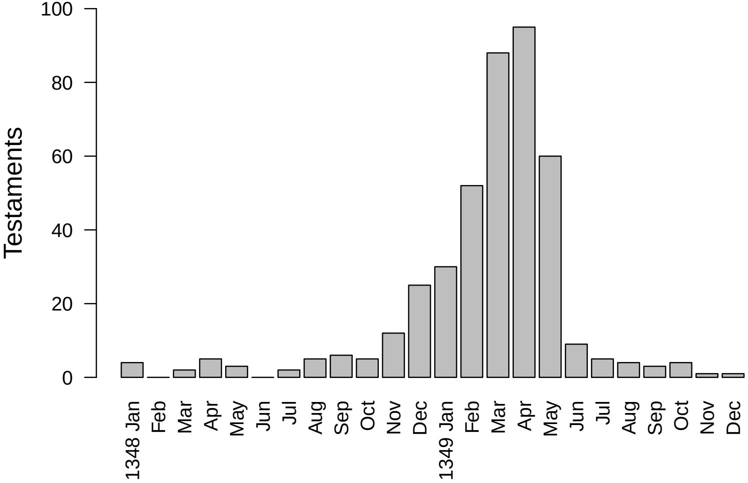


$N = 249$ (of 250 reported)

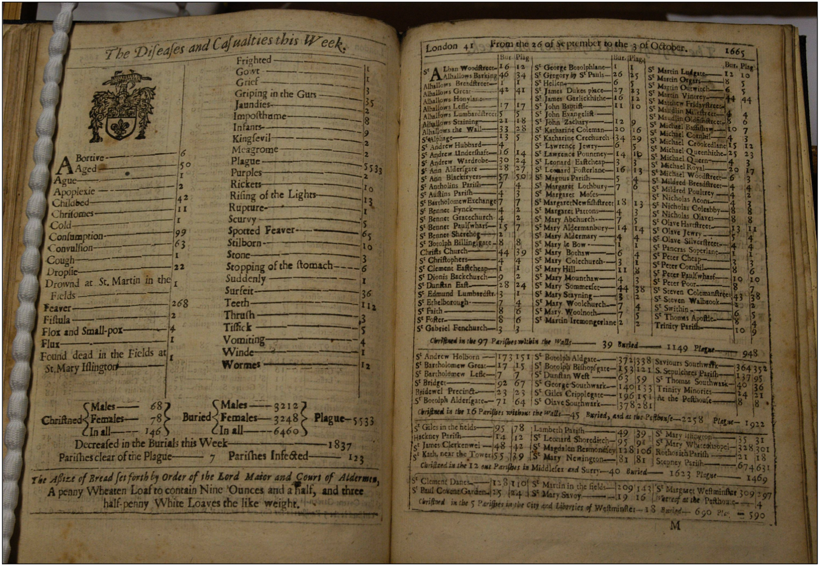
Some SARS Facts

- High case fatality
  - 1918 flu $< 3\%$
  - SARS $> 10\%$
- Long hospital stays
  - Mean time from admission to discharge or death: ~ 25 days in Hong Kong
- 8098 probable cases, 774 deaths
- How bad would it have been if it had not been controlled?

The Black Death in London, England, 1348–1349



London Bill of Mortality, 26 Sept to 3 Oct 1665



The Diseases and Casualties this Week.

Abortive	6	Frighted	1
Aged	50	Gowt	1
Ague	1	Grief	1
Apoplexie	2	Griping in the Guts	35
Childbed	42	Jaundies	2
Chrisomes	11	Impostume	8
Cold	1	Infants	9
Consumption	99	Kingsevil	2
Convulsion	63	Meagrome	2
Cough	1	Plague	5533
Dropsie	22	Purples	2
Drownd at St. Martin in the Fields	1	Rickets	10
Feaver	268	Rising of the Lights	13
Fistula	2	Rupture	1
Flox and Small-pox	4	Scurvy	5
Flux	1	Spotted Feaver	65
Found dead in the Fields at St. Mary Islington	1	Stilborn	10
		Stone	3
		Stopping of the stomach	6
		Suddenly	1
		Surfeit	36
		Teeth	112
		Thrush	3
		Tissick	5
		Vomiting	4
		Winde	1
		Wormes	12

Christned: Males 68, Females 78, In all 146
Buried: Males 3212, Females 3248, In all 6460
Plague 5533

Decreased in the Burials this Week 1837
Parishes clear of the Plague 7 Parishes Infected 123

The Assize of Bread set forth by Order of the Lord Maior and Court of Aldermen, A penny Wheaten Loaf to contain Nine Ounces and a half, and three half-penny White Loaves the like weight.

London 41 From the 26 of September to the 3 of October. 1665

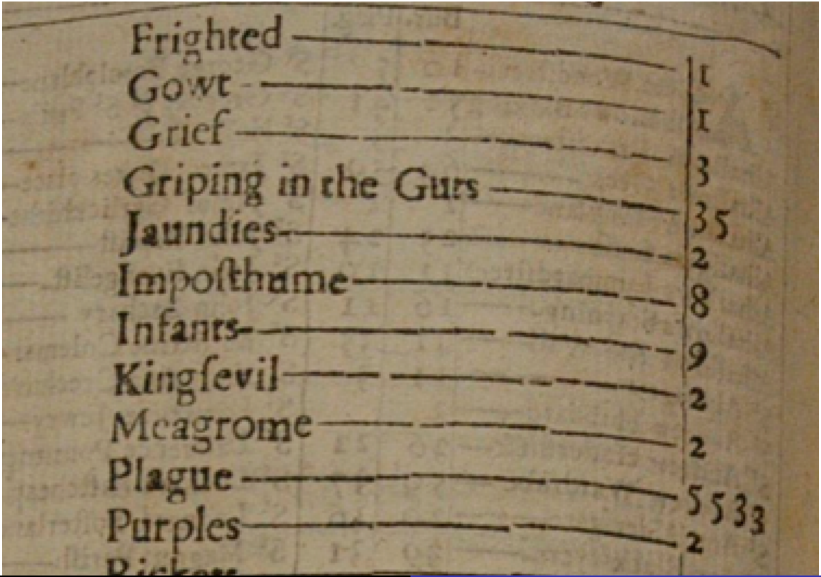
Christned in the 97 Parishes within the Walls — 39 Buried — 1149 Plague — 948

Christned in the 16 Parishes without the Walls — 45 Buried, and at the Pesthouse — 2258 Plague — 1922

Christned in the 12 out Parishes in Middlesex and Surry — 40 Buried — 1623 Plague — 1469

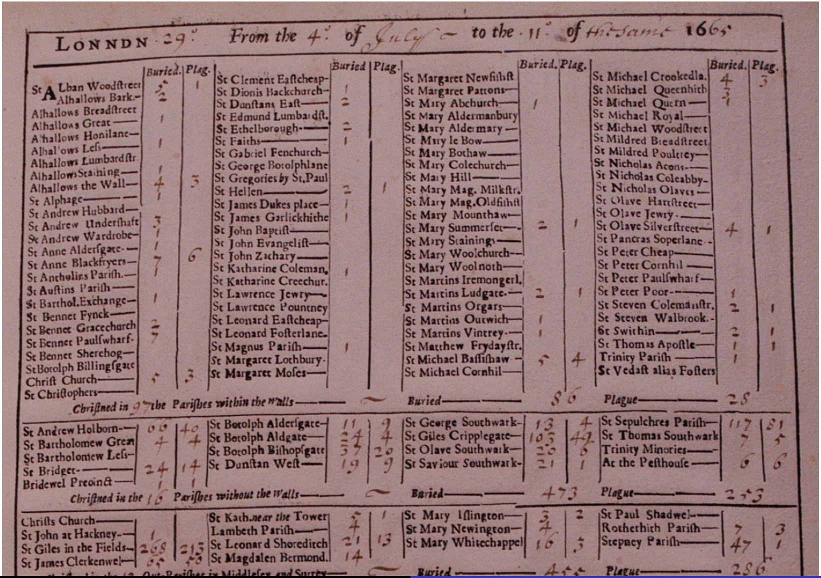
Christned in the 5 Parishes in the City and Liberties of Westminster — 18 Buried — 690 Plague — 590

London Bill of Mortality, 26 Sept to 3 Oct 1665



Frighted	1
Gowt	1
Grief	3
Griping in the Guts	35
Jaundies	2
Impoſthume	8
Infants	9
Kingſevil	2
Meagrome	2
Plague	5533
Purples	2

Mortality Bills are typically handwritten



LONNDN 29 From the 4 of July to the 11 of the same 1665

	Buried	Plag.		Buried	Plag.		Buried	Plag.		Buried	Plag.
St Alban Woodstreet	2	1	St Clement Eastcheap	1		St Margaret Newfishst			St Michael Crookedla.	4	3
Alhallows Bark.	2		St Dionis Backchurch	2		St Margaret Pattons			St Michael Queenhith	3	
Alhallows Breadstreet			St Dunstans East			St Mary Abchurch	1		St Michael Quern	1	
Alhallows Great	1		St Edmund Lumbardst.			St Mary Aldermanbury			St Michael Royal		
Alhallows Honilane			St Ethelborough	2		St Mary Aldermary			St Michael Woodstreet		
Alhallows Less	1		St Faiths	1		St Mary le Bow			St Mildred Breadstreet		
Alhallows Lumbardstr.			St Gabriel Fenchurch			St Mary Bothaw			St Mildred Poultrey		
Alhallows Staining	1		St George Botolphlane			St Mary Colechurch			St Nicholas Acons		
Alhallows the Wall	4	3	St Gregories by St.Paul			St Mary Hill			St Nicholas Coleabby		
St Alphage	1		St Hellen	2	1	St Mary Mag. Milkstr.			St Nicholas Olaves		
St Andrew Hubbard			St James Dukes place	1		St Mary Mag.Oldfishst			St Olave Hartstreet		
St Andrew Undershaft	3		St James Garlickhithe	1		St Mary Mounthaw			St Olave Jewry		
St Andrew Wardrobe	1		St John Baptist			St Mary Summerset	2	1	St Olave Silverstreet	4	1
St Anne Aldersgate	1		St John Evangelist			St Mary Stainings			St Pancras Soperlane		
St Anne Blackfryers	7	6	St John Zachary			St Mary Woolchurch			St Peter Cheap		
St Antholins Parish	1		St Katharine Coleman	1		St Mary Woolnoth			St Peter Cornhil		
St Austins Parish			St Katharine Creechur.			St Martins Iremongerl.			St Peter Paulswharf		
St Barthol.Exchange	1		St Lawrence Jewry			St Martins Ludgate	2	1	St Peter Poor	1	
St Bennet Fynck			St Lawrence Pountney			St Martins Orgars			St Steven Colemanstr.	2	1
St Bennet Gracechurch	2		St Leonard Eastcheap			St Martins Outwich	1		St Steven Walbrook		
St Bennet Paulswharf	7		St Leonard Fosterlane			St Martins Vintrey	1		St Swithin	2	1
St Bennet Sherehog			St Magnus Parish	1		St Matthew Frydaystr.			St Thomas Apostle	1	1
StBotolph Billingsgate			St Margaret Lothbury			St Michael Bassishaw	5	4	Trinity Parish	1	
Christ Church	5	3	St Margaret Moses			St Michael Cornhil			St Vedast alias Fosters		
St Christophers											

Christned in the 97 Parishes within the Walls — Buried — 86 Plague — 28

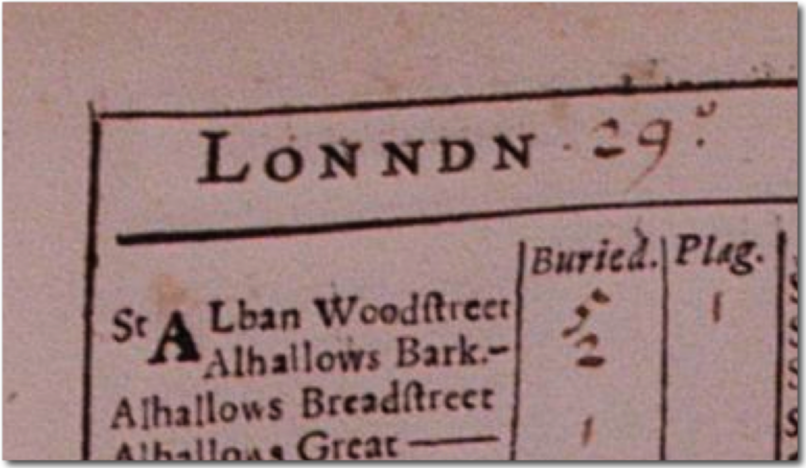
	Buried	Plag.		Buried	Plag.		Buried	Plag.		Buried	Plag.
St Andrew Holborn	66	40	St Botolph Aldersgate	11	9	St George Southwark	13	4	St Sepulchres Parish	117	81
St Bartholomew Great	4	4	St Botolph Aldgate	24	4	St Giles Cripplegate	103	42	St Thomas Southwark	7	5
St Bartholomew Less			St Botolph Bishopsgate	37	20	St Olave Southwark	20	6	Trinity Minories		
St Bridget	24	14	St Dunstan West	19	9	St Saviour Southwark	21	1	At the Pesthouse	6	6
Bridewel Precinct	1	1									

Christned in the 16 Parishes without the Walls — Buried — 473 Plague — 253

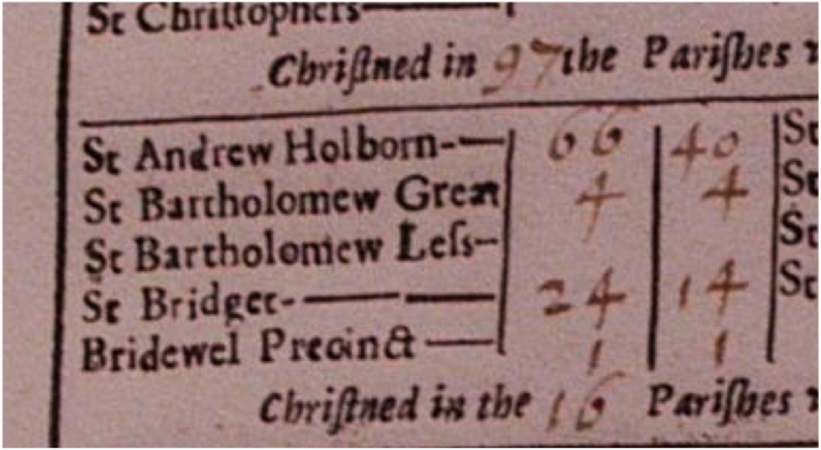
	Buried	Plag.		Buried	Plag.		Buried	Plag.		Buried	Plag.
Christs Church			St Kath. near the Tower	5	1	St Mary Islington	3	2	St Paul Shadwel		
St John at Hackney	1		Lambeth Parish	4		St Mary Newington	4		Rotherhith Parish	7	3
St Giles in the Fields	268	213	St Leonard Shoreditch	21	13	St Mary Whitechappel	16	5	Stepney Parish	47	1
St James Clerkenwel	65	58	St Magdalen Bermond.	14							

Christned in the 12 Out-Parishes in Middlesex and Surrey — Buried — 455 Plague — 256

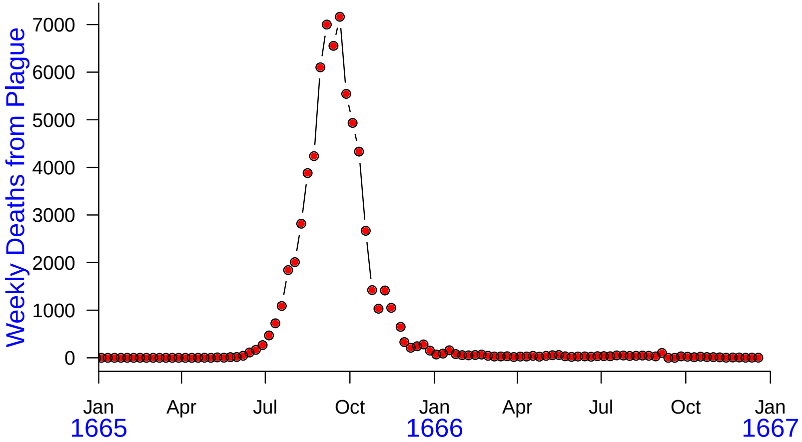
But handwriting is usually very clear



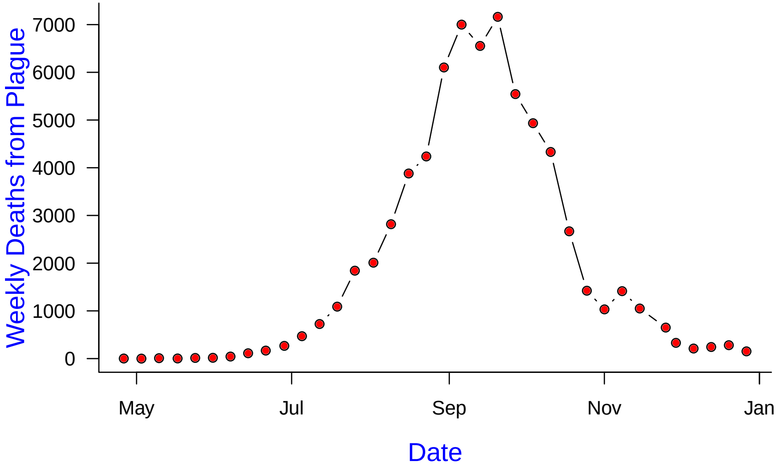
But handwriting is usually very clear



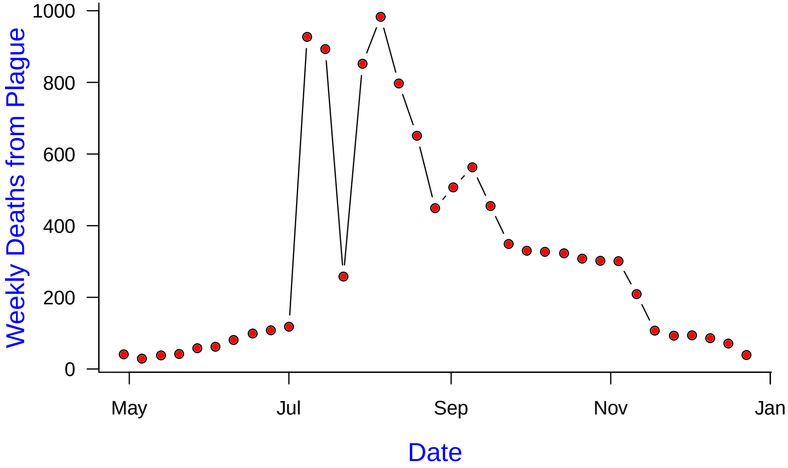
The Great Plague of London, 1665



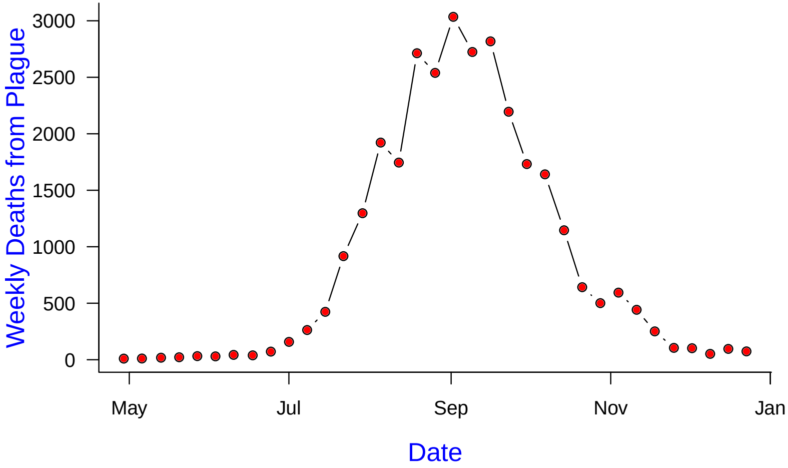
The Great Plague of London, 1665



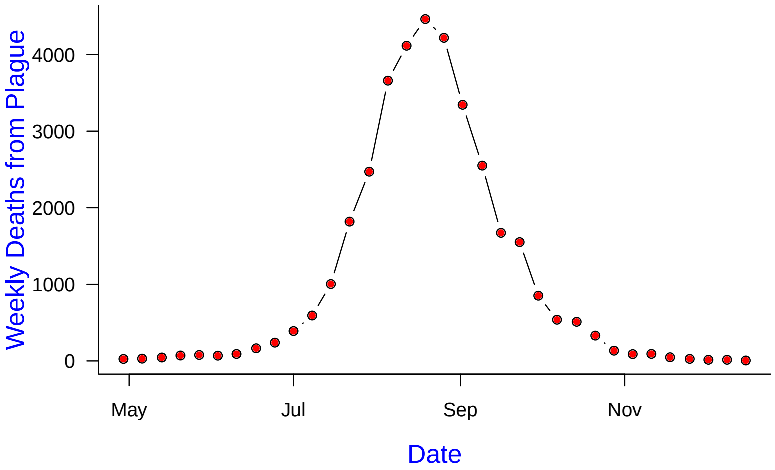
London Plague of 1593



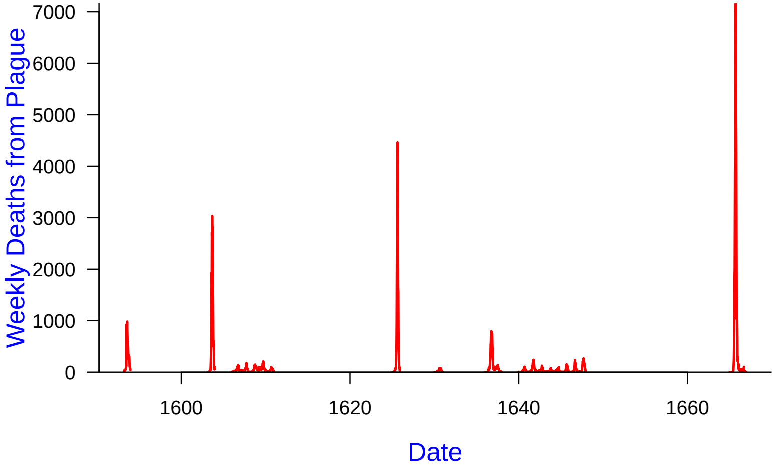
London Plague of 1603



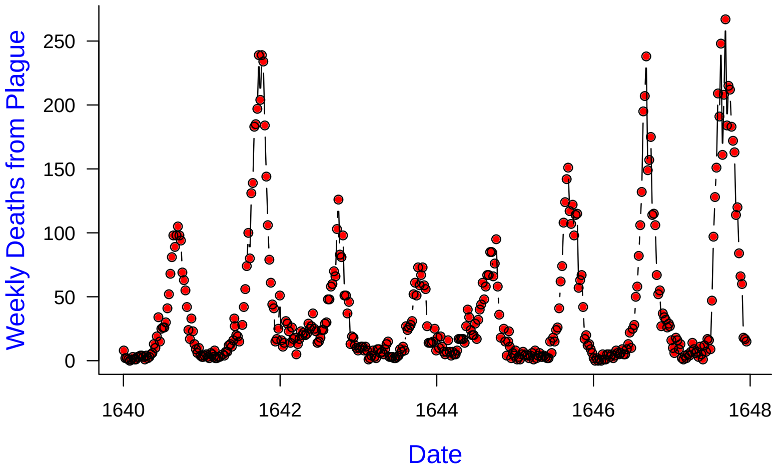
London Plague of 1625



Weekly Deaths from Plague in London, 1592–1666



Weekly Plague in London, 1640–1648



Some Plague Facts

- Plague epidemics recorded from Roman times to early 1900s.
- $\gtrsim 1/3$ Europe's population died in "Black Death" of 1348
  - ~ 300 years for the population to reach the same level.
- Recently (2011) established (at McMaster!) that the pathogen that caused The Black Death was *Yersinia pestis*

  [Bos *et al.* 2011, ***Nature*** **478**, 506–510]
- More recently (2014) established (again at McMaster!) that the pathogen that caused The Plague of Justinian (541–543 AD) was *Yersinia pestis*

  [Wagner *et al.* 2014, ***Lancet Infectious Diseases*** **14**, 319–326]
- *Y. pestis* still a concern?
  Yes: Rodent reservoir, antibiotic-resistant strains, bioterrorism
- **Spatial data** for any plagues? Yes, for London in 1665. . .

Visualization of spatial structure of Great Plague

- GIS encoding of parish boundaries
- Overlay parish boundaries on more modern map for reference
- Colour parishes as they become infected
- Is there evidence for spatial spread or was the spatial pattern random?
- DE low-tech animation...
- CBC high-tech animation...
  - *The Nature of Things*, 21 August 2014. http://www.cbc.ca/natureofthings/episodes/secrets-in-the-bones-the-hunt-for-the-black-death-killer

Please consider...

5 minute *Student Respiratory Illness Survey:*

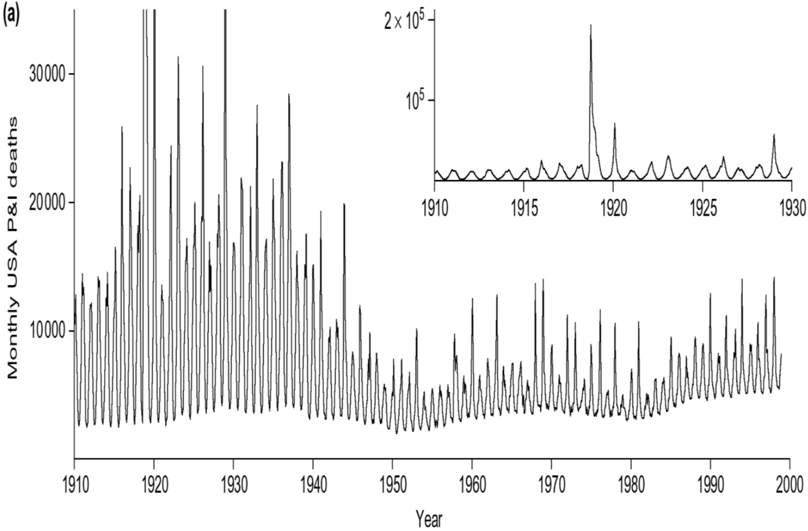
https://surveys.mcmaster.ca/limesurvey/index.php/893454

> *Please complete this anonymous survey to help us monitor the patterns of respiratory illness, over-the-counter drug use, and social contact within the McMaster community. There are no risks to filling out this survey, and your participation is voluntary. You do not need to answer any questions that make you uncomfortable, and all information provided will be kept strictly confidential. Thanks for participating.*
>
> *–Dr. Marek Smieja (Infectious Diseases)*

Visualization of entire course of the Great Plague

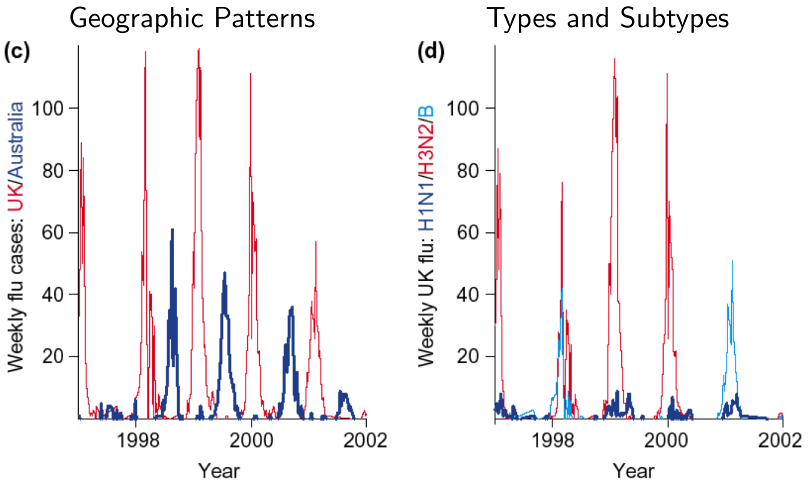
- What happenned after initial spatial spread?
- Visualize full spatial epidemic structure
- Show magnitude of epidemic in each parish with cylinder.
- Epidemic Visualization (`EpiVis`) software by Junling Ma.

P&I mortality in U.S.A., 1910–1998



Earn, Dushoff & Levin 2002, *Trends in Ecology and Evolution* **17**, 334–340

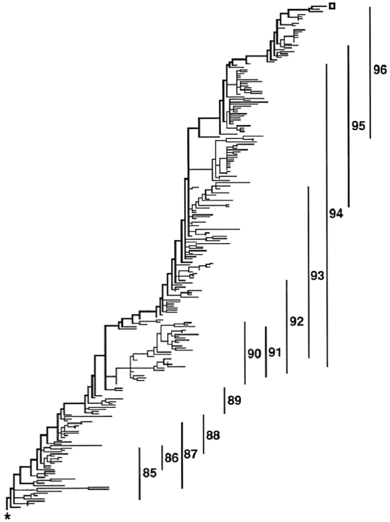
Influenza Incidence Patterns (lab confirmed)



Earn, Dushoff & Levin 2002, *Trends in Ecology and Evolution* **17**, 334–340

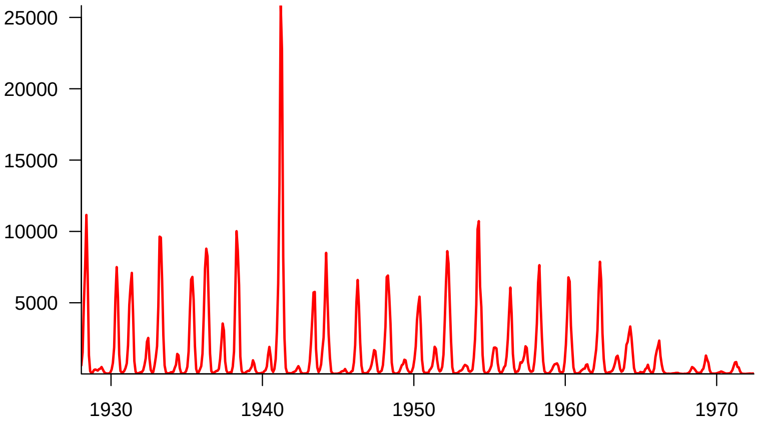
Influenza Evolution

Molecular phylogenetic reconstruction of influenza A/H3N2 evolution, 1985–1996 (Fitch *et al.* 1997)



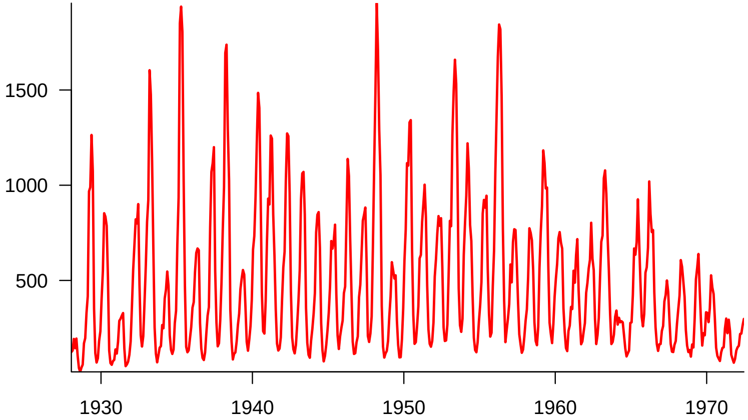
Measles in New York City, 1928–1972

Monthly Cases

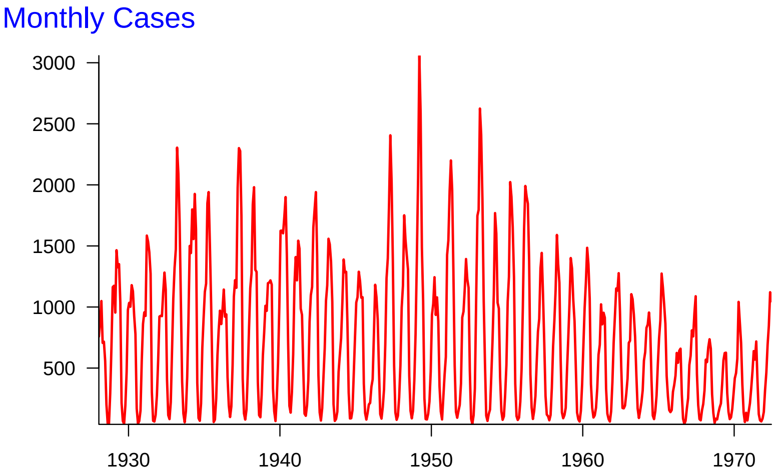


Mumps in New York City, 1928–1972

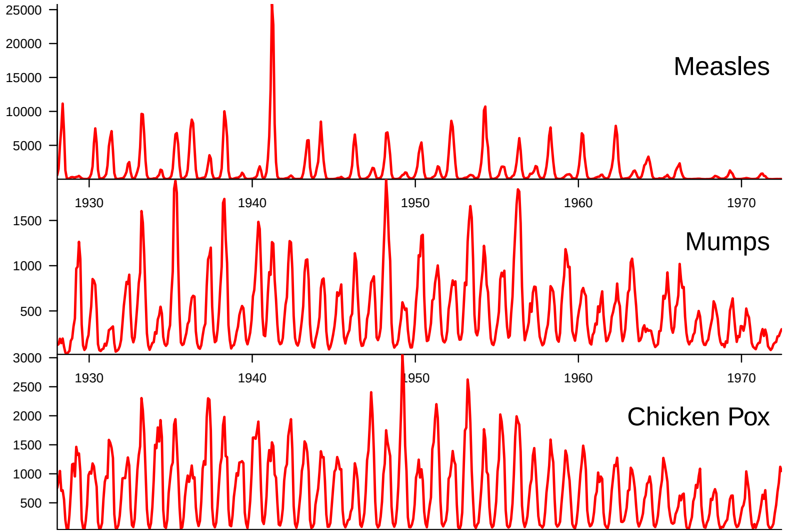
Monthly Cases



Chicken Pox in New York City, 1928–1972

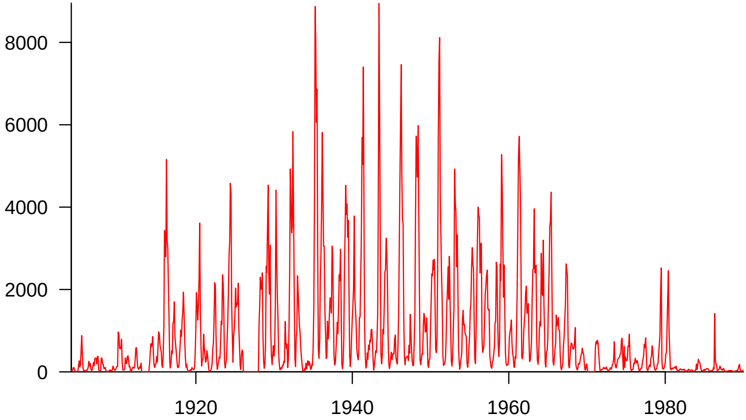


Childhood diseases in New York City, 1928–1972



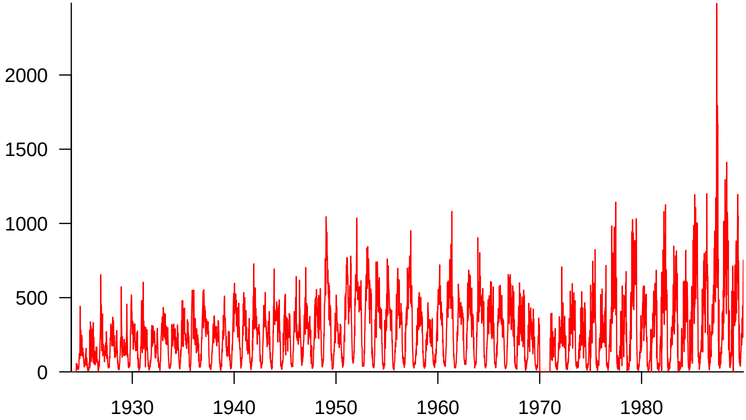
Measles in Ontario, 1904–1989

Monthly Cases

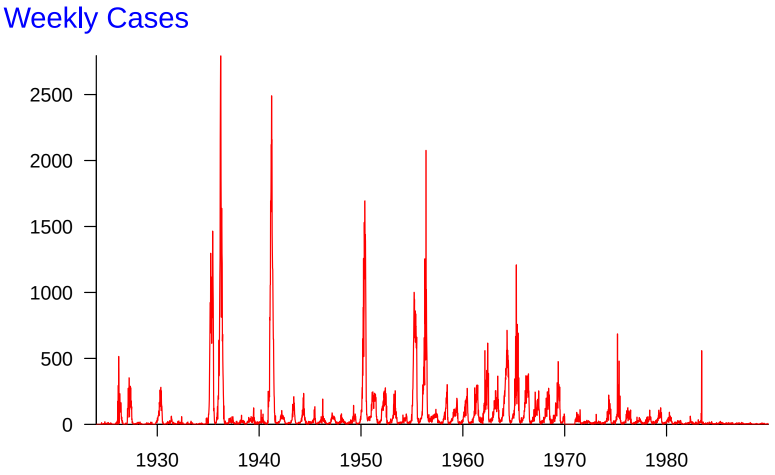


Chicken Pox in Ontario, 1924–1989

Monthly Cases

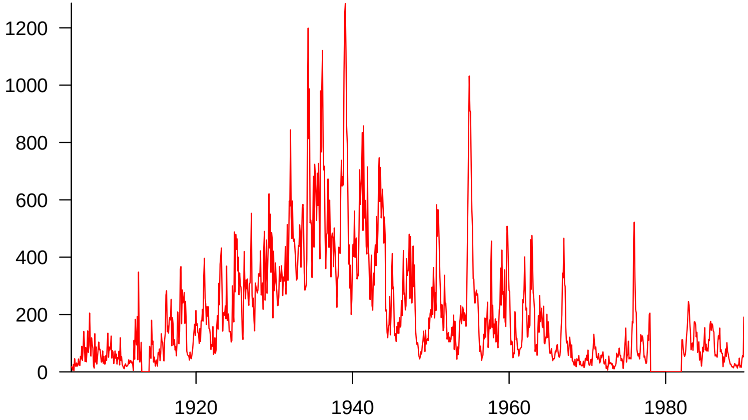


Rubella in Ontario, 1924–1989

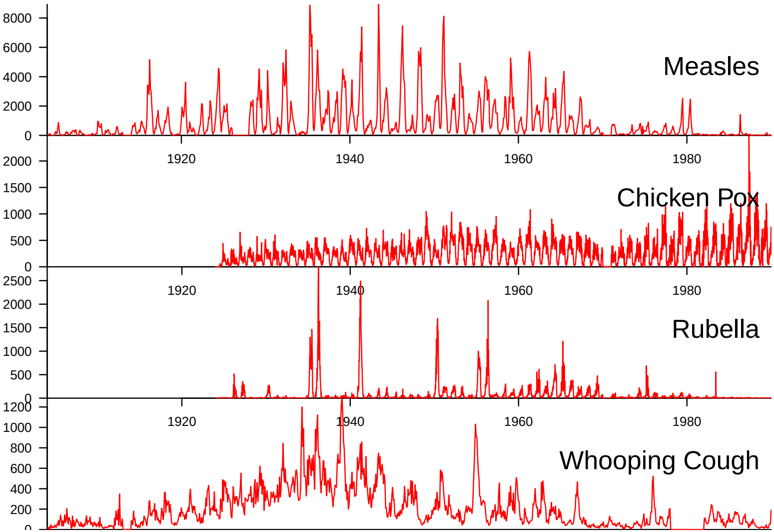


Whooping Cough in Ontario, 1904–1989

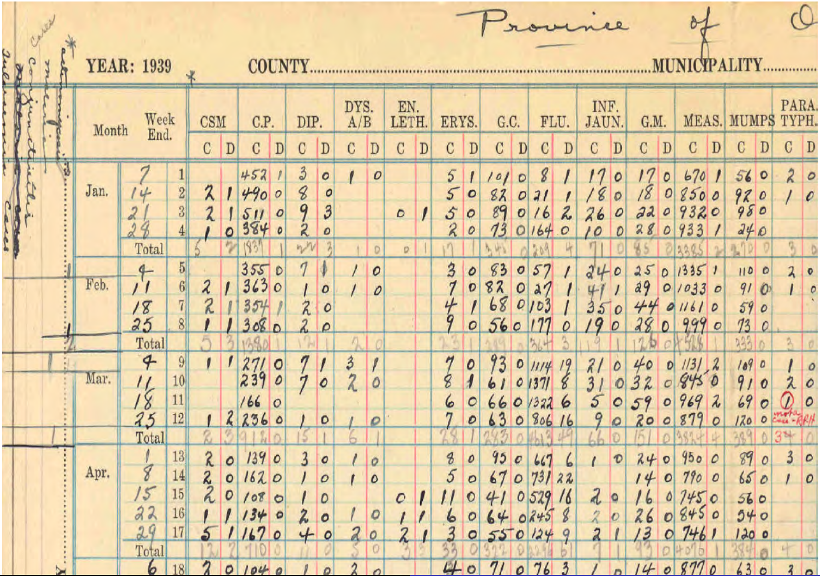
Monthly Cases



Childhood diseases in Ontario, 1904–1989



Ontario Disease Notification Data



Province of O

YEAR: 1939 COUNTY........ MUNICIPALITY........

Month	Week End.		CSM C	CSM D	C.P. C	C.P. D	DIP. C	DIP. D	DYS. A/B C	DYS. A/B D	EN. LETH. C	EN. LETH. D	ERYS. C	ERYS. D	G.C. C	G.C. D	FLU. C	FLU. D	INF. JAUN. C	INF. JAUN. D	G.M. C	G.M. D	MEAS. C	MEAS. D	MUMPS C	MUMPS D	PARA. TYPH. C	PARA. TYPH. D
Jan.	7	1			452	1	3	0	1	0			5	1	101	0	8	1	17	0	17	0	670	1	56	0	2	0
	14	2	2	1	490	0	8	0					5	0	82	0	21	1	18	0	18	0	850	0	92	0	1	0
	21	3	2	1	511	0	9	3			0	1	5	0	89	0	16	2	26	0	22	0	932	0	98	0		
	28	4	1	0	384	0	2	0					2	0	73	0	164	0	10	0	28	0	933	1	24	0		
	Total		5	2	1839	1	22	3	1	0	0	1	17	1	345	0	209	4	71	0	85	0	3385	2	270	0	3	0
Feb.	4	5			355	0	7	1	1	0			3	0	83	0	57	1	24	0	25	0	1335	1	110	0	2	0
	11	6	2	1	363	0	1	0	1	0			7	0	82	0	27	1	41	1	29	0	1033	0	91	0	1	0
	18	7	2	1	354	1	2	0					4	1	68	0	103	1	35	0	44	0	1161	0	59	0		
	25	8	1	1	308	0	2	0					9	0	56	0	177	0	19	0	28	0	999	0	73	0		
	Total		5	3	1380	1	12	1	2	0			23	1	289	0	364	3	119	1	126	0	4528	1	333	0	3	0
Mar.	4	9	1	1	271	0	7	1	3	1			7	0	93	0	1114	19	21	0	40	0	1131	2	109	0	1	0
	11	10			239	0	7	0	2	0			8	1	61	0	1371	8	31	0	32	0	845	0	91	0	2	0
	18	11			166	0							6	0	66	0	1322	6	5	0	59	0	969	2	69	0	1	0
	25	12	1	2	236	0	1	0	1	0			7	0	63	0	806	16	9	0	20	0	879	0	120	0	mota case - RRH	
	Total		2	3	912	0	15	1	6	1			28	1	283	0	4613	49	66	0	151	0	3824	4	389	0	3+	0
Apr.	1	13	2	0	139	0	3	0	1	0			8	0	95	0	667	6	1	0	24	0	950	0	89	0	3	0
	8	14	2	0	162	0	1	0	1	0			5	0	67	0	731	22			14	0	790	0	65	0	1	0
	15	15	2	0	108	0	1	0			0	1	11	0	41	0	529	16	2	0	16	0	745	0	56	0		
	22	16	1	1	134	0	2	0	1	0	1	1	6	0	64	0	245	8	2	0	26	0	845	0	54	0		
	29	17	5	1	167	0	4	0	2	0	2	1	3	0	55	0	124	9	2	1	13	0	746	1	120	0		
	Total		12	2	710	0	11	0	5	0	3	3	33	0	322	0	2296	61	7	1	93	0	4076	1	384	0	4	0
	6	18	2	0	104	0	1	0	2	0			4	0	71	0	76	3	1	0	14	0	877	0	63	0	3	0

Dominion Bureau of Statistics Disease Notification Data

VITAL STATISTICS BRANCH - COMMUNICABLE DISEASE SECTION

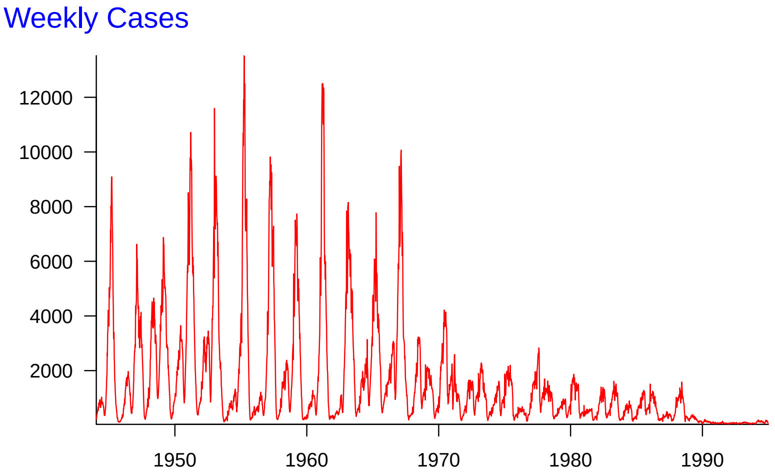
Cases of Whooping Cough Reported by Provincial Health Departments, Year 1924

WEEK ENDING	P.E.I.		N.S.		N.B.		QUE.		ONT.		MAN.		SASK.		ALTA.		B.C.		CANADA	
	wk	mt	wk	mt	wk	mt	wk	mt	wk	mt	wk	mt	wk	mt	wk	mt	wk	mt	wk	mt
1 Jan 5			11										1						12	
2 12			29										18						47	
3 19			37										32						69	
4 26			75	152				68		181		36	13	64		97		4	88	602
5 Feb 2			12		1								53						66	
6 9			5										40						45	
7 16			31										14						45	
8 23			- 2	50	1	2		267		202		48	4	111		116		1	7	797
9 Mar 1			2										21						23	
10 8													9						9	
11 15			3										11						14	
12 22			60										34						94	
13 29			2	61				144		140		52	15	90		15		7	17	515
14 Apr 5			9										11						20	
15 12			1										12						13	
16 19			26		1								8						35	
17 26			14	50	3	4		42		140		39	16	47		67		5	33	394
18 May 3			26										2						28	

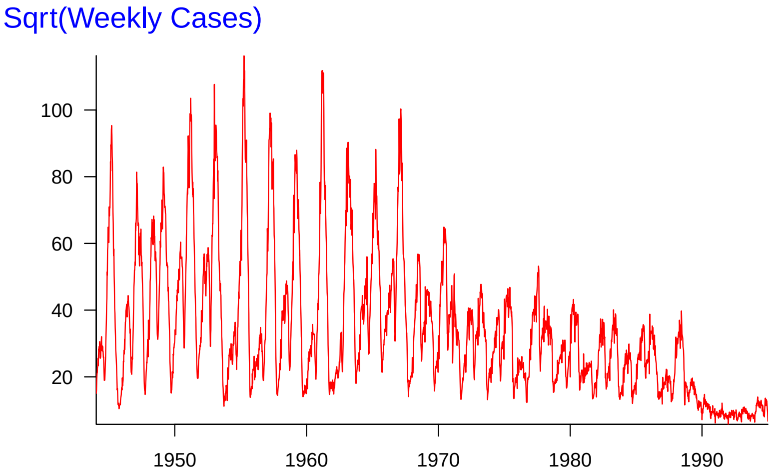
Recurrent epidemics of childhood infections

- Childhood diseases in New York City, 1928–1972
- Childhood diseases in Ontario, 1904–1989

Measles incidence in England and Wales, 1944–1995



Measles incidence in England and Wales, 1944–1995



Why study measles epidemics?

- In 2017, $\sim 110,000$ deaths from measles
- A major cause of *vaccine-preventable* deaths.
- Potential impact in developed countries during vaccine scares (e.g., MMR scare in UK in 1990s).

- Understand past patterns
- Predict future patterns
- Manipulate future patterns
- Develop vaccination strategy that can. . .

Other reasons to model infectious disease epidemics

- Mathematical models make hypotheses and inferences precise
  - Give better advice to policymakers
  - Make better predictions
- Host-pathogen dynamics are important aspects of ecosystem dynamics
  - Infectious disease models more likely to be successful than predator-prey models
- Excellent data for human infectious diseases
  - Models can be tested!

Modelling population dynamics of childhood infections

- The basic SIR model cannot explain recurrent epidemics.
- What should we do?. . . The usual options:
  1. Get depressed, drop the course.
  2. Keep developing models until we can explain recurrent epidemics.
- First, let's talk about tools that allow us to make our questions about time series data more precise.

Please consider. . .

5 minute *Student Respiratory Illness Survey:*

https://surveys.mcmaster.ca/limesurvey/index.php/893454

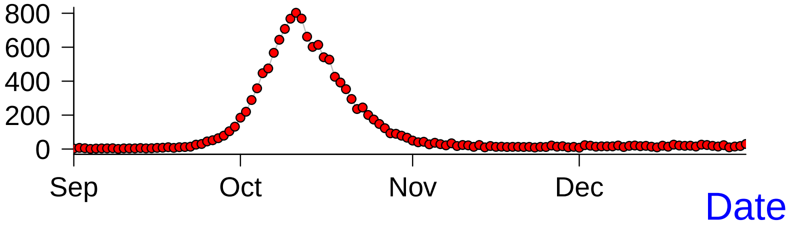
Please complete this anonymous survey to help us monitor the patterns of respiratory illness, over-the-counter drug use, and social contact within the McMaster community. There are no risks to filling out this survey, and your participation is voluntary. You do not need to answer any questions that make you uncomfortable, and all information provided will be kept strictly confidential. Thanks for participating.

–Dr. Marek Smieja (Infectious Diseases)

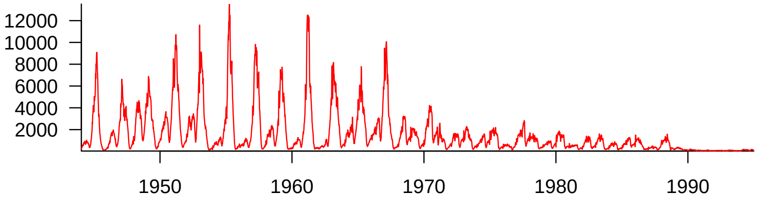
Epidemic Data Analysis

Time Plots of Temporal Epidemic Patterns

1918 P&I

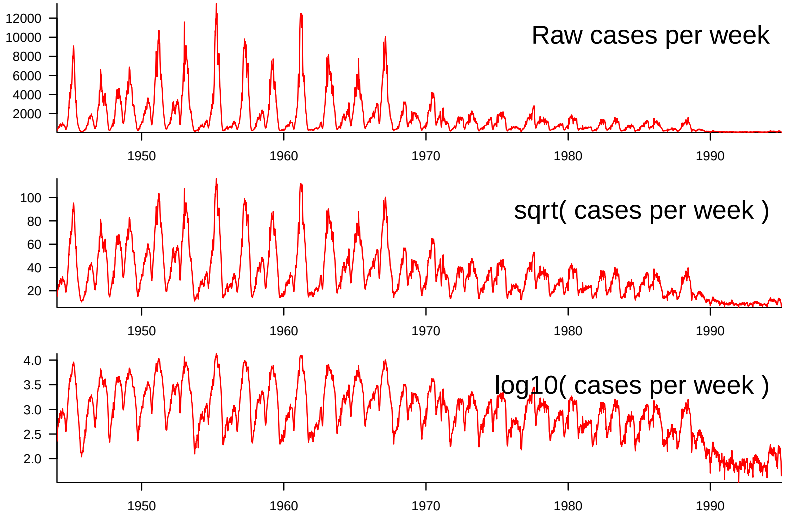


Weekly Measles in England and Wales



Time Plots of Transformed Data

- Reveal unobvious aspects of time series



Times Plots of Smoothed Data

- Reveal trends clouded by noise or seasonality
- *Moving Average:*

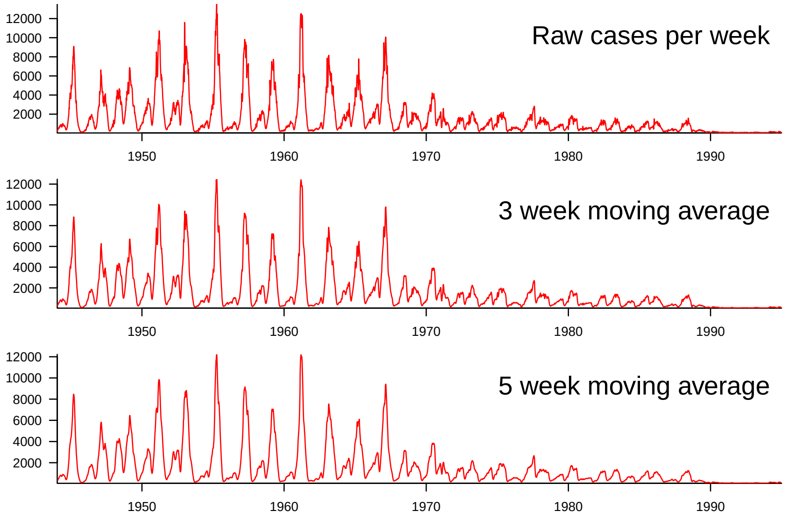
$$x_t \to \frac{1}{2a+1} \sum_{i=-a}^{a} x_{t+i}$$

  - Replace original data points x_t with averages of nearby points.
- *Linear filter:*

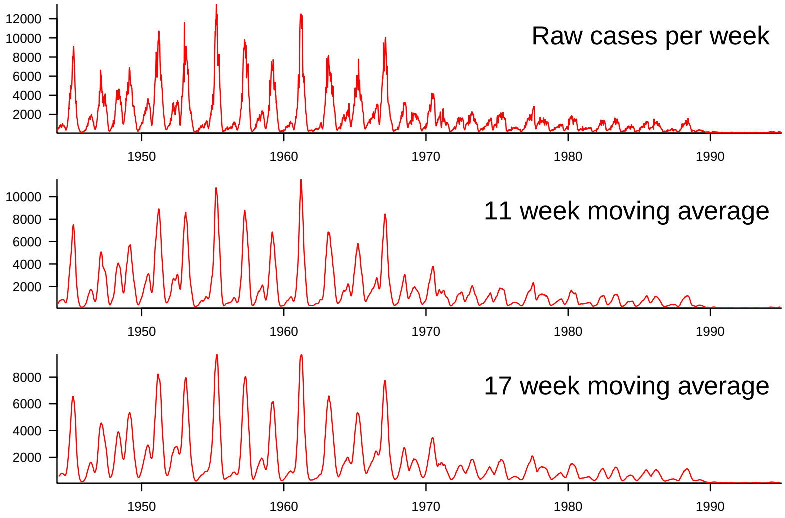
$$x_t \to \sum_{i=-\infty}^{\infty} \lambda_i x_{t+i}$$

  - Generalization of moving average.
  - *Weights* λ_i can be nonlinear functions of i.

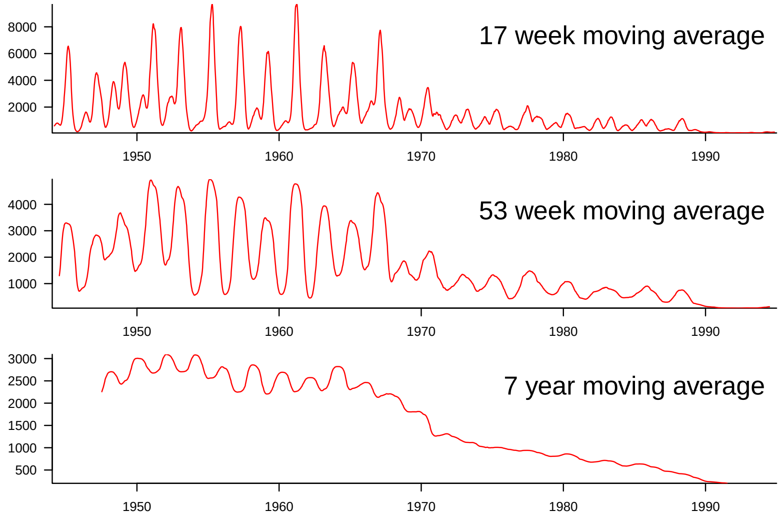
Times Plots of Smoothed Data



Times Plots of Smoothed Data



Times Plots of Smoothed Data



Correlation

- Recurrent epidemics $\implies$ number of cases now is correlated with number of cases in the past and the future.
- Given N pairs of observations of different quantities, $\{(x_i, y_i) : i = 1, \ldots, N\}$, the *correlation coefficient* is defined to be

$$r = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}}$$

where $\bar{x}$ and $\bar{y}$ are the means of $\{x_i\}$ and $\{y_i\}$, respectively.

Correlation

Properties of the correlation coefficient:

- $-1 \leq r \leq 1$ (Proof? Cauchy-Schwarz inequality)
- $r = 1 \iff$ all points lie on a line with positive slope ("complete positive correlation")
- $r = -1 \iff$ all points lie on a line with negative slope ("complete negative correlation")
- $r \simeq 0 \implies$ "uncorrelated"
- *Interpretation:* r^2 is the proportion of the variance in y explained by a linear function of x.

Derivations and discussions:

- MathWorld on r^2, Wikipedia on r^2
- Wikipedia on general coefficient of determination

Autocorrelation

- Given a single sequence of observations $\{x_t : t = 1, \ldots, N\}$, we can compute the correlation of each observation with the observation k time steps in the future.

- Thus, we consider the pairs of observations $\{(x_t, x_{k+t}) : t = 1, \ldots, N-k\}$ and define the *autocorrelation coefficient at lag k* to be

$$r_k = \frac{\sum_{t=1}^{N-k} (x_t - \bar{x}_{1,N-k})(x_{k+t} - \bar{x}_{k+1,N})}{\sqrt{\sum_{t=1}^{N-k} (x_t - \bar{x}_{1,N-k})^2 \sum_{t=1}^{N-k} (x_{k+t} - \bar{x}_{k+1,N})^2}}$$

where $\bar{x}_{1,N-k}$ and $\bar{x}_{k+1,N}$ are the means of first and last $N-k$ observations, respectively.

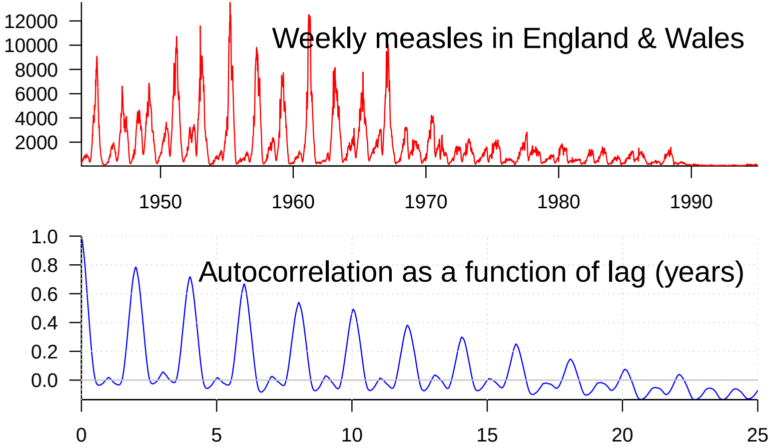
Autocorrelation

- If number of observations N is large and lag $k \ll N$ then

$$r_k \simeq \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{k+t} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}$$

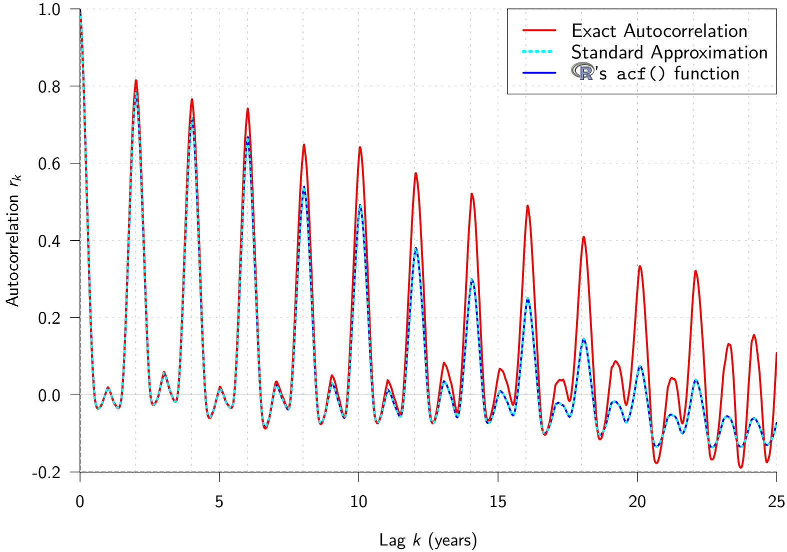
- Approximation of r_k is worse for larger lags k
- Plot of autocorrelation r_k as a function of lag k is called the *correlogram*.

Correlogram



- Peaks in correlogram $\Longrightarrow$ periodicities in original time series.
- Correlograms of temporal segments are often informative.

Correlogram: exact vs. approximate r_k



Spectral Density

- Can we compute the dominant periods in the time series? (Rather than estimating them by eye from the correlogram.)
- Express the time series as a Fourier series:

$$x_t = a_0 + \left(\sum_{p=1}^{(N/2)-1} \left(a_p \cos \omega_p t + b_p \sin \omega_p t \right) \right) + a_{N/2} \cos \pi t \,,$$

where $\omega_p = 2\pi p/N$.

- Compute the *Fourier coefficients* $\{a_p\}$, $\{b_p\}$ by taking inner products with $\cos \omega_p t$ and $\sin \omega_p t$.

Spectral Density

- Fourier coefficients of x_t are:

$$a_0 = \bar{x} = \frac{1}{N}\sum_t x_t \,,$$

$$a_p = \frac{2}{N}\sum_t x_t \cos\omega_p t \,, \qquad b_p = \frac{2}{N}\sum_t x_t \sin\omega_p t \,,$$

$$a_{N/2} = \frac{1}{N}\sum_t (-1)^t x_t \,,$$

where sum is over observation times.

- Estimated power spectral density (PSD) at frequency ω_p is*:

$$I(\omega_p) = \frac{N}{4\pi}(a_p^2 + b_p^2)$$

* The normalization by $N/4\pi$ is the convention chosen by Chatfield (2004, "Analysis of Time Series: An Introduction"). Other normalization conventions are also in common use.

Please consider. . .

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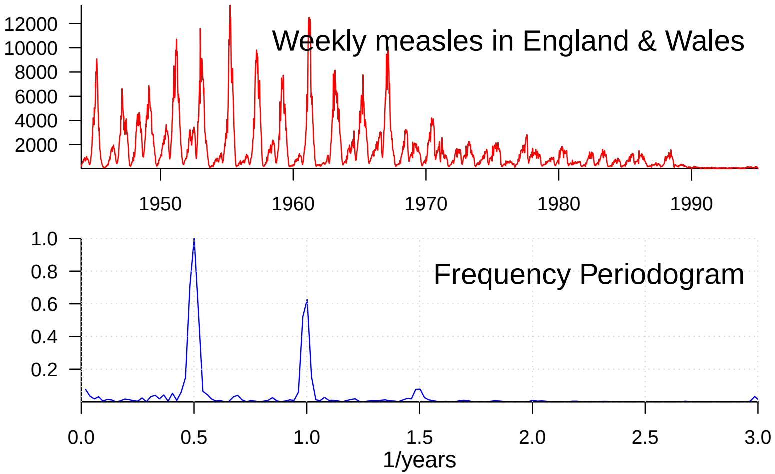
Spectral Density

- There are many different ways to express the power spectral density (aka *power spectrum*).
- Most common/useful equivalence is that the power spectrum is the discrete Fourier transform of the correlogram:

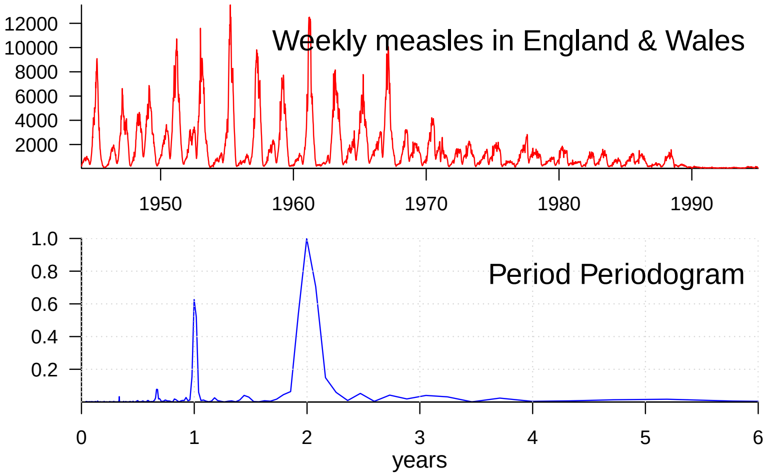
$$I(\omega_p) = \frac{1}{\pi}\left(r_0 + 2\sum_{k=1}^{N-1} r_k \cos \omega_p k\right)$$

- Plot of estimated power spectrum as a function of frequency ω_p is called the *frequency periodogram* or just the *periodogram*.

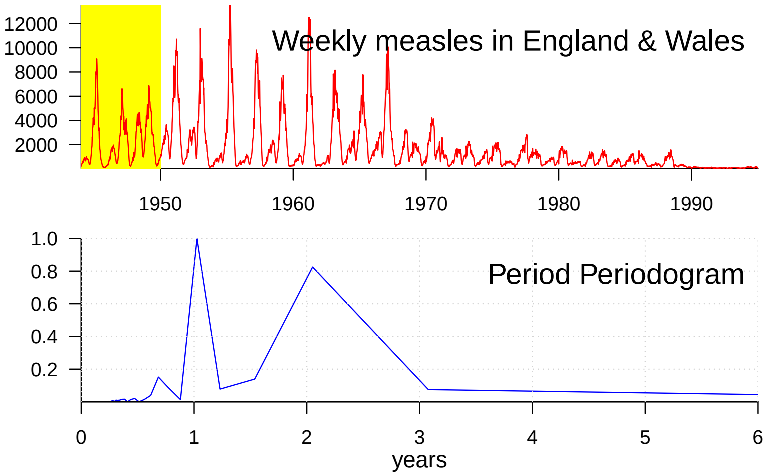
Spectral Density



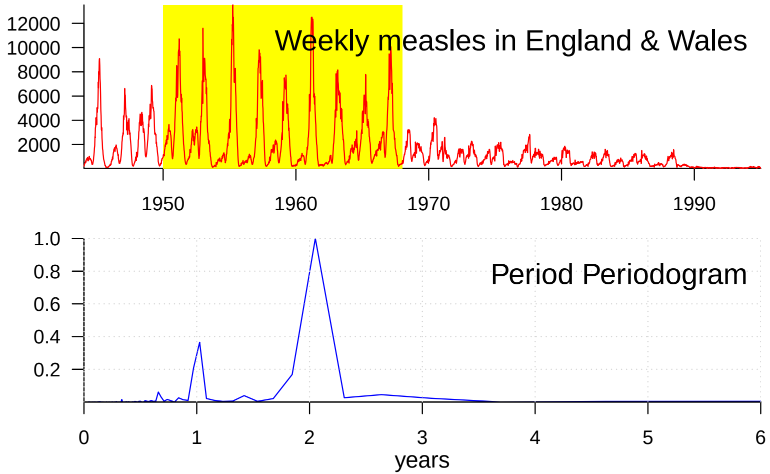
Spectral Density



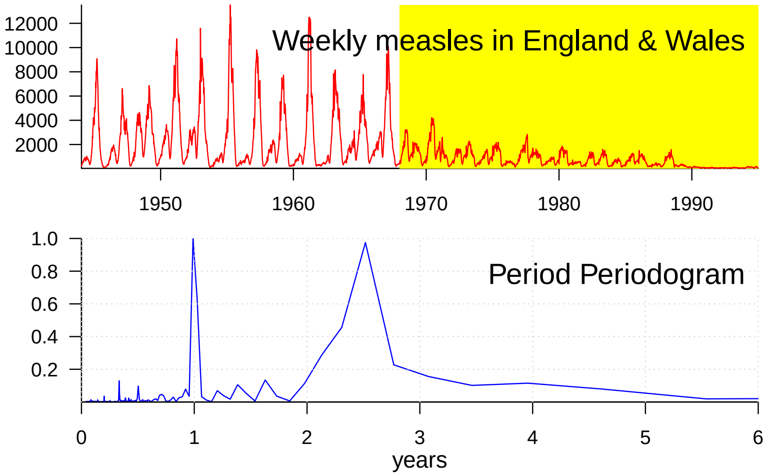
Spectral Density of Temporal Segments



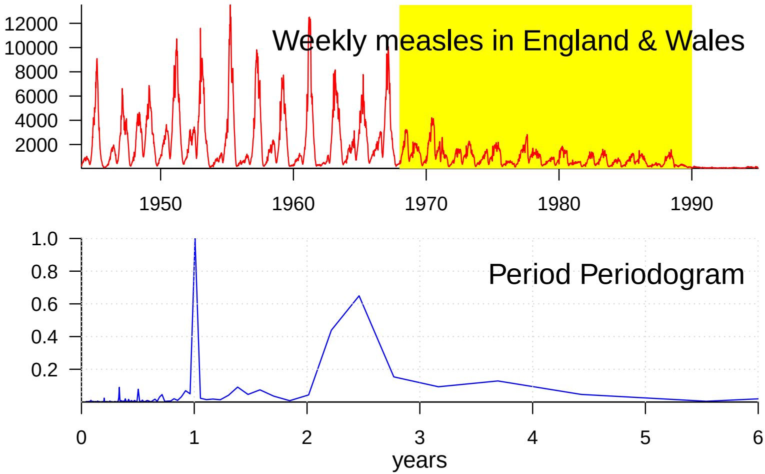
Spectral Density of Temporal Segments



Spectral Density of Temporal Segments



Spectral Density of Temporal Segments



Spectral Density Properties

- Periodogram is discrete Fourier transform of correlogram
- Same information in correlogram and periodogram
- Periodogram usually easier to interpret
- In R, calculate power spectrum with `spectrum()`
- The power spectrum $I(\omega_p)$ partitions the variance in the time series with respect to frequency ω_p.
  - Parseval's theorem implies $\frac{1}{N}\sum_t (x_t - \bar{x})^2 = \frac{1}{2\pi N}\sum_{p>0} I(\omega_p)$. But $\frac{1}{N}\sum_t (x_t - \bar{x})^2 = \mathrm{Var}\{x_t\}$, hence $I(\omega_p)/(2\pi N)$ is the proportion of the variance in the time series associated with period $2\pi/\omega_p$. [For details, see Chatfield (2004).]

Basic Time Series Analysis of Epidemic Data