Student Name: ____________________ Student Number: _______________

Mathematics 3A03 — Real Analysis I

TERM TEST #1 — 23 October 2017

Duration: 90 minutes

Notes:

- No calculators, notes, scrap paper, or aids of any kind are permitted.
- This test consists of 8 **pages** (*i.e.*, **4 double-sided pages**). There are **9 questions** in total. Bring any discrepancy to the attention of your instructor or invigilator.
- All questions are to be answered on this test paper. The final page is blank to provide extra space if needed.
- The first 6 questions do not require any justification for your answers. For these, you will be assessed on your answers only. *Do not justify your answers to these questions.*
- Always write clearly. An answer that cannot be deciphered cannot be marked.
- The marking scheme is indicated in the margin. The maximum total mark is 50.

Question	**Mark**
1	
2	
3	
4	
5	
Subtotal	

Question	**Mark**
6	
7	
8	
9	
Subtotal	

Total	

Student Name: ____________________ Student Number: _______________

GOOD LUCK and ENJOY!

MARKS

[1] **QUESTION 1.** *In order to obtain any credit for this question, both parts must be answered in clear handwriting at the top of every page of this test.*

(a) What is your name?

(b) What is your student number?

[5] **QUESTION 2.** (*Circle the correct answer.*) For each of the following sets, determine whether it is ***Countable*** or ***Uncountable***. *Do not justify your answers.*

(a) The interval $(2, 4)$

Countable **Uncountable**

(b) $\mathbb{Z} \times \mathbb{N}$

Countable **Uncountable**

(c) $\mathbb{R} \times \mathbb{R}$

Countable **Uncountable**

(d) $\{x \in (0, 1) \ : \ x \notin \mathbb{Q}\}$

Countable **Uncountable**

(e) $\left\{m + n\pi : m, n \in \mathbb{Q}\right\}$

Countable **Uncountable**

Student Name: ___________________ Student Number: ______________

[4] **QUESTION 3.** (*Circle the correct answer.*) Determine whether each the following statements is ***True*** or ***False***. *Do not justify your answers.*

(a) Every non-empty subset of $\mathbb{N}$ has a least element.

True **False**

(b) There are Cauchy sequences of real numbers that do not converge.

True **False**

(c) For every $x > 0$, there is a positive rational number $q \in \mathbb{Q}$ so that $q < x$.

True **False**

(d) Every monotone sequence converges.

True **False**

[4] **QUESTION 4.** (*Circle the correct answer.*) Determine whether each the following statements is ***Never True***, ***Sometimes True***, or ***Always True***. *Do not justify your answers.*

(a) If $\{x_n\}$ and $\{y_n\}$ are both convergent sequences, and $y_n \neq 0$ for all n, then $\{x_n/y_n\}$ is a convergent sequence.

Never True **Sometimes True** **Always True**

(b) If $A_1, A_2, \ldots$ are open subsets of $\mathbb{R}$, then the intersection $\cap_{n\in\mathbb{N}} A_n$ is open.

Never True **Sometimes True** **Always True**

(c) Suppose $E \subset \mathbb{R}$. Then the closure of the interior of E is equal to E.

Never True **Sometimes True** **Always True**

(d) Suppose $\{x_n\}$ is a bounded, decreasing sequence. Then $\{x_n\}$ has exactly one convergent subsequence.

Never True **Sometimes True** **Always True**

Student Name: ____________________ Student Number: _______________

[6] **QUESTION 5.** For each of the sets E in the table below, answer ***YES*** or ***NO*** in each column to indicate whether or not E is open, bounded or dense in $\mathbb{R}$. *Do not justify your answers.*

Set E	**Open?**	**Bounded?**	**Dense in** $\mathbb{R}$**?**
$(\sqrt{2}, 3+\sqrt{2}]$			
$\{m + n\pi : m, n \in \mathbb{Q}\}$			

[6] **QUESTION 6.** For each of the sets E in the table below, fill in the associated point or set in each column, *i.e.*, for each set E state the greatest lower bound ($\inf(E)$), the closure ($\overline{E}$), and the set of accumultation points (E'). If the requested point or set does not exist, then indicate this with the symbol $\nexists$. *Do not justify your answers.*

Set E	$\inf(E)$	$\overline{E}$	E'
$(\sqrt{2}, 3+\sqrt{2}) \cap \mathbb{Q}$			
$\left\{\frac{n+1}{n} : n \in \mathbb{N}\right\}$			

[12] **QUESTION 7.**

(a) Let $\{s_n\}$ be a sequence. State the formal definition of "the sequence $\{s_n\}$ converges as $n \to \infty$".

(b) Suppose $\{x_n\}$ is a bounded sequence. Use the formal definition to prove that the sequence $\left\{\dfrac{x_n}{n^2+1}\right\}$ converges as $n \to \infty$.

[6] **QUESTION 8.** Show that if $S \subseteq \mathbb{R}$ is open and non-empty, then S is uncountable. (You may use, without proof, the fact that the interval $(0, 1)$ is uncountable.)

[6] **QUESTION 9.** Suppose $S \subseteq \mathbb{R}$ is a closed, bounded, and non-empty set of real numbers. Show that S contains its supremum: $\sup(S) \in S$.

THE END