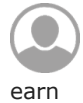


$$\int_M d\omega = \int_{\partial M} \omega$$

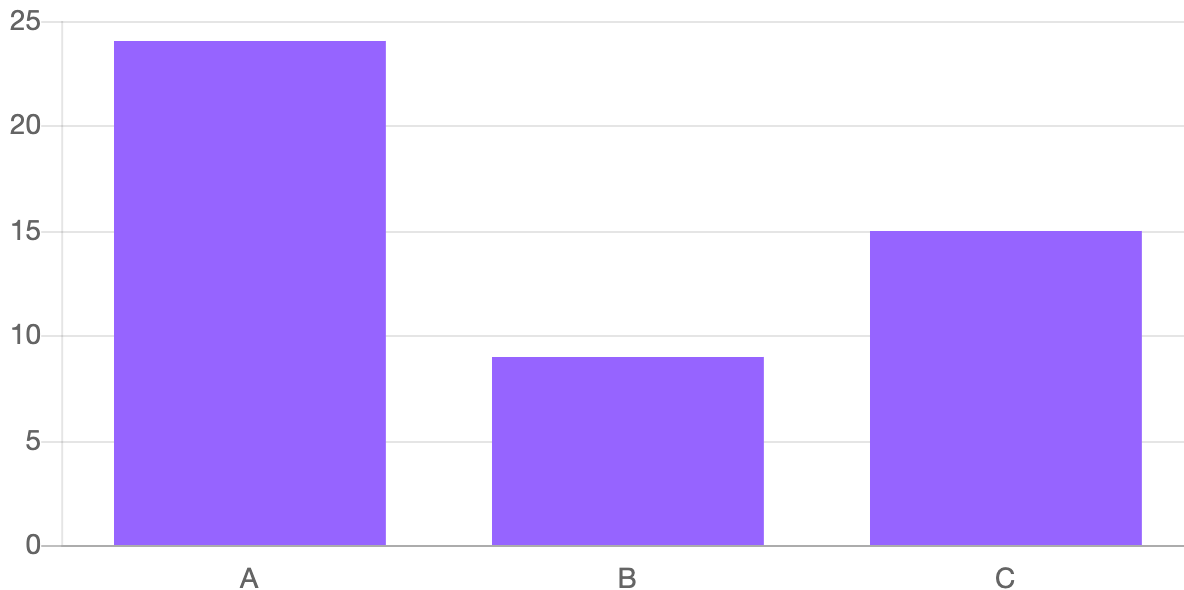


## Metric spaces: Set differences of balls

**Question #1** Suppose  $0 < r_1 < r_2$  and  $B_{r_1}(x)$  and  $B_{r_2}(x)$  are balls centered at  $x$  in the metric space  $(\mathbb{R}, \text{standard})$ .

Which of the following are true?

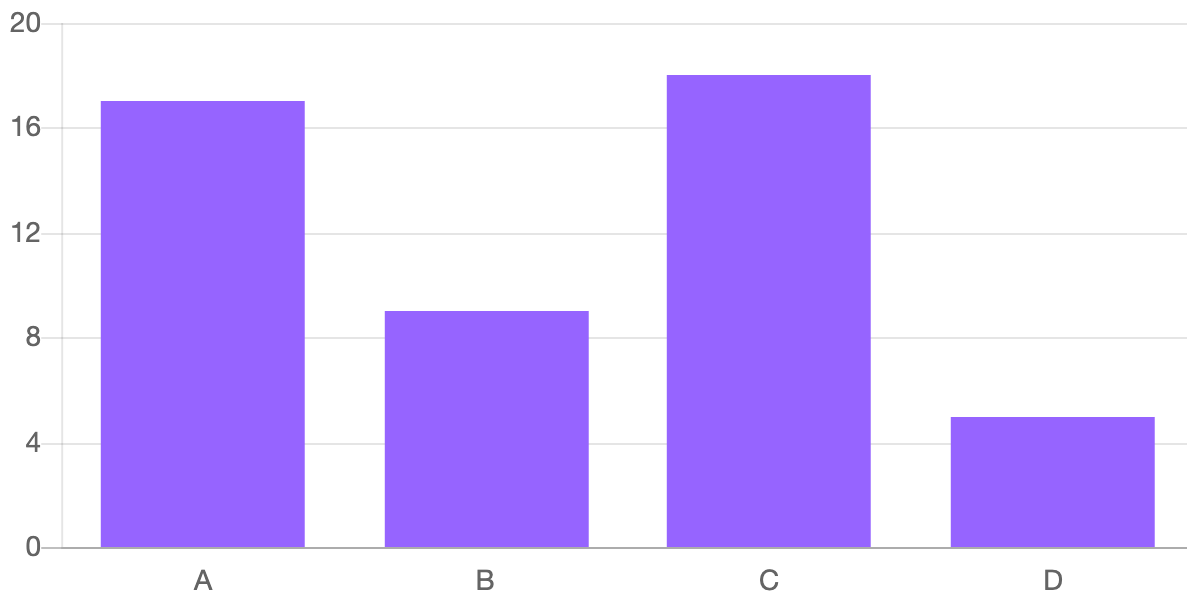
- (A)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is open;
- (B)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is closed;
- (C)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is neither open nor closed.



**Question #2** Now suppose  $0 < r_1 < r_2$  and  $B_{r_1}(x)$  and  $B_{r_2}(x)$  are balls centered at  $x$  in the metric space  $(\mathbb{R}^n, \text{Euclidean})$  for  $n \geq 2$ .

Which of the following are true?

- (A)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is open;
- (B)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is closed;
- (C)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is neither open nor closed;
- (D) It depends on the dimension  $n$ .



**Question #3** Now suppose  $0 < r_1 < r_2$  and  $B_{r_1}(x)$  and  $B_{r_2}(x)$  are balls centered at  $x$  in the metric space  $(\mathbb{R}^n, \text{discrete})$ .

Which of the following are true?

- (A)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is open;
- (B)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is closed;
- (C)  $B_{r_2}(x) \setminus B_{r_1}(x)$  is neither open nor closed;
- (D) It depends on the dimension  $n$ ;
- (E) It depends on the specific values of  $r_1$  and  $r_2$ .

