childsmath $\int_M d\omega = \int_{\partial M} \omega$

Math 3A03
Poll Results

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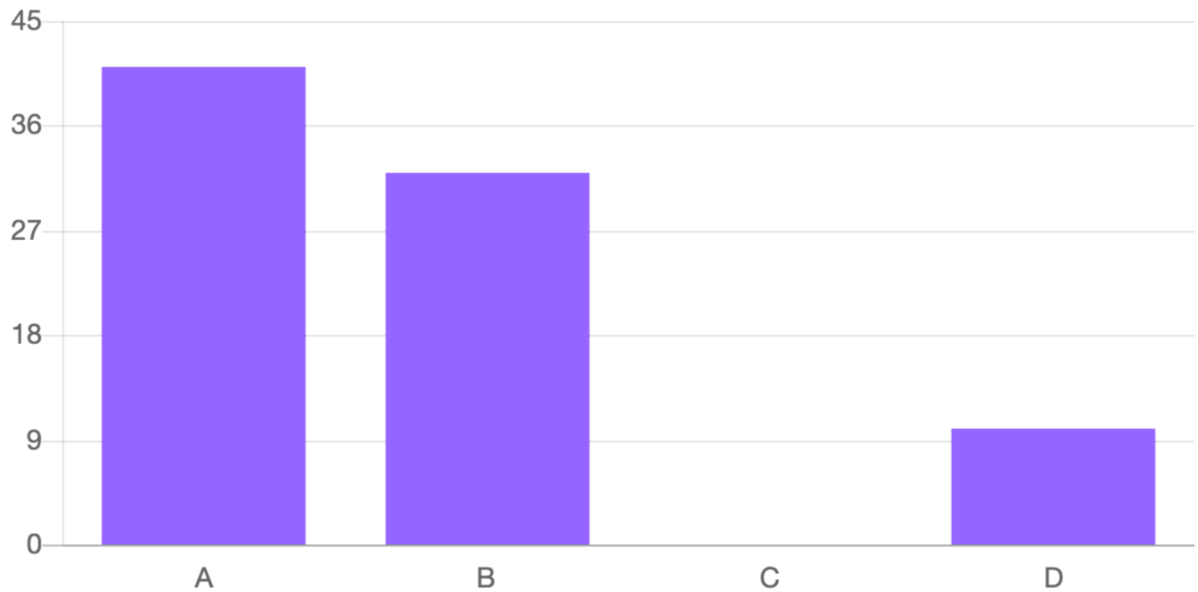
Assignment 5: Metric spaces

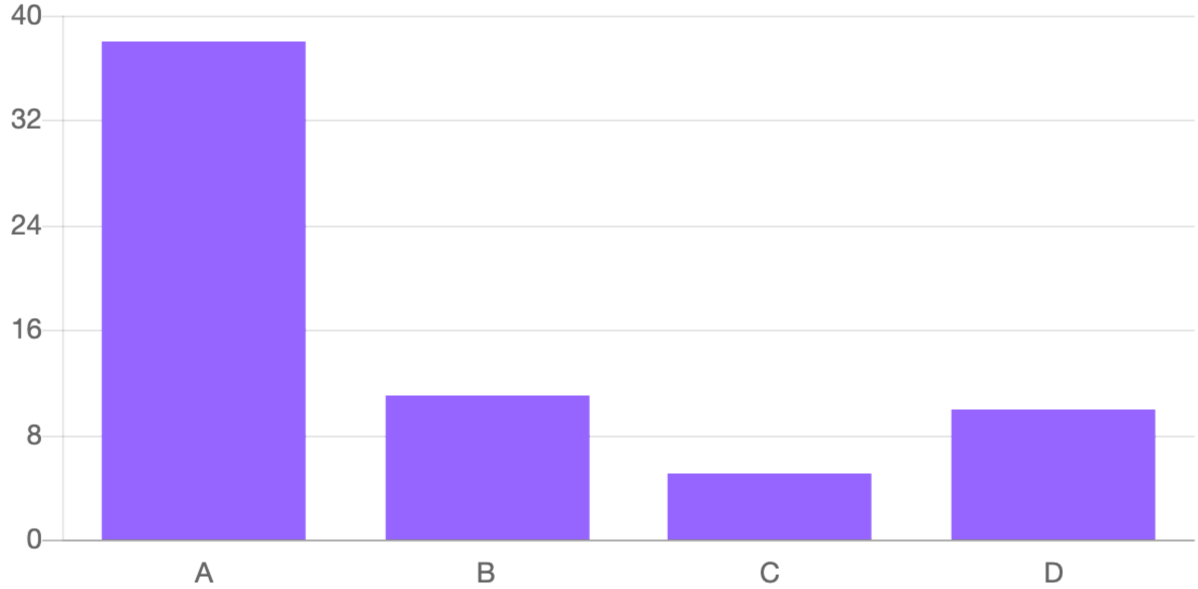
Question #1 Suppose $x = (x_1, \ldots, x_N) \in \mathbb{R}^N$ and $x_n = (x_{n,1}, \ldots, x_{n,N}) \in \mathbb{R}^N$ for each $n \in \mathbb{N}$. Which of the following statements are true for the sequence $(x_n)_{n\in\mathbb{N}}$ in the metric space $(\mathbb{R}^N, \text{Euclidean})$?

(A) $x_n \to x \quad \Longrightarrow \quad x_{n,j} \to x_j \quad \text{in } (\mathbb{R},\text{standard}) \quad \forall j = 1, 2, \ldots, N;$
(B) $x_n \to x \quad \Longleftarrow \quad x_{n,j} \to x_j \quad \text{in } (\mathbb{R},\text{standard}) \quad \forall j = 1, 2, \ldots, N;$
(C) (x_n) never converges;
(D) I have not had sufficient time to think about this yet.



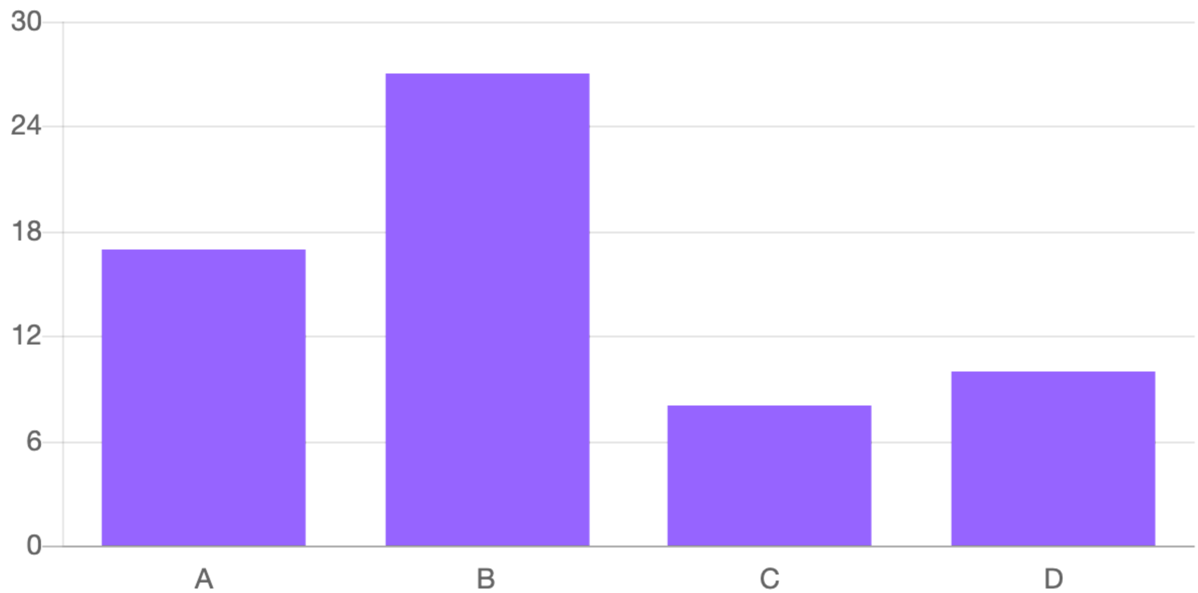
Question #2 Let V be an inner product space, with norm $\|v\| = \sqrt{\langle v, v\rangle}$, $v \in V$. Assume that the sequences $(v_n)_{n\in\mathbb{N}}$, $(w_n)_{n\in\mathbb{N}}$ are both convergent, $v_n \to v$ and $w_n \to w$. Which of the following statements are true?

(A) $\langle v_n, w_n\rangle \to \langle v, w\rangle$, in $(\mathbb{R}$,standard);
(B) $\langle v_n, w_n\rangle \to \|v\| + \|w\|$, in $(\mathbb{R}$,standard);
(C) $\langle v_n, w_n\rangle$ does not necessarily converge in $(\mathbb{R}$,standard);
(D) I have not had sufficient time to think about this yet.



Question #3 In a metric space $(\mathcal{M}, d)$, any set with no limit points is:

(A) open;
(B) closed;
(C) neither open nor closed;
(D) I have not had sufficient time to think about this yet.



Question #4 By $C^n[a,b]$ we mean the space of n-times continuously differentiable functions on the closed interval $[a,b]$, i.e., functions that have n derivatives and that the nth derivative is continuous on $[a,b]$. For $n=0$, we mean $C[a,b]$. The derivative operator on $C^n[a,b]$ for $n>0$ is defined by $(D(f))(x)=f'(x)$. The sup norm $\|\cdot\|_\infty$ on $C[a,b]$ is still a norm on $C^n[a,b]$ for $n>0$ (why?). If $n>0$, we can also define a "derivative norm",

$$\|f\|_D = \|f\|_\infty + \|f'\|_\infty.$$

Which of the following statements are true?

(A) $D : C^1[a,b] \to C[a,b]$ is a continuous operator under the sup norm;

(B) $D : C^1[a,b] \to C[a,b]$ is a continuous operator under the derivative norm;

(C) $D : C^1[a,b] \to C[a,b]$ is not a continuous operator under any norm;

(D) I have not had sufficient time to think about this yet.