childsmath $\int_M d\omega = \int_{\partial M} \omega$

Math 3A03
Poll Results

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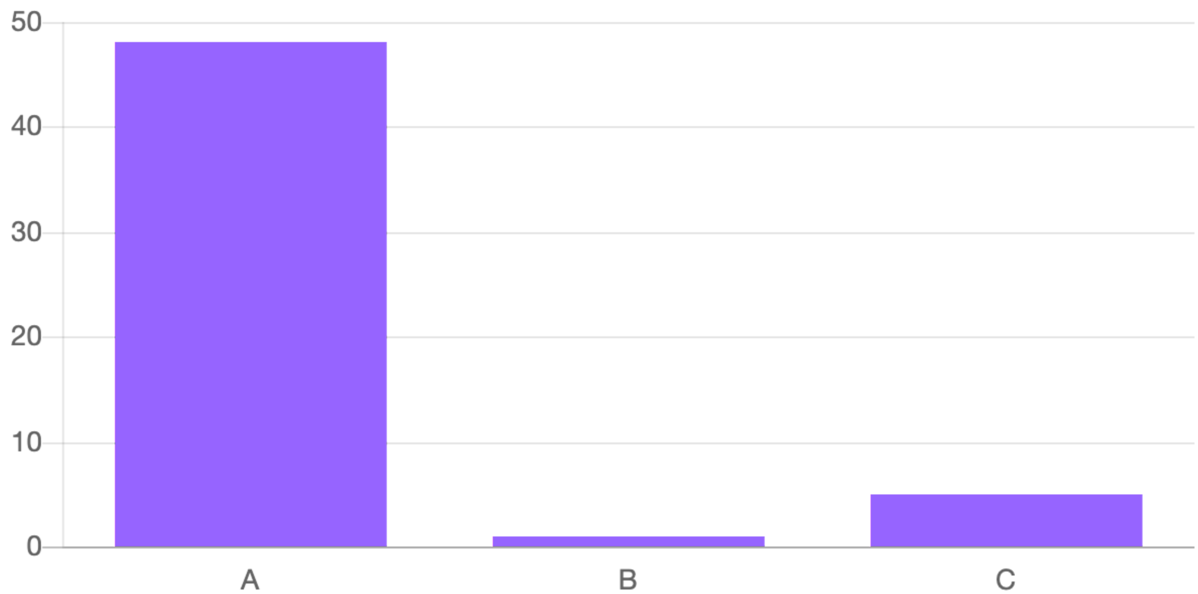
Assignment 2: The Integral

Question #1 Suppose $a < b$ and $f : [a, b] \to \mathbb{R}$ is integrable on the closed interval $[a, b]$. Then:

(A) f is necessarily integrable on any closed subinterval of $[a, b]$;
(B) There might exist a closed subinterval of $[a, b]$ on which f is not integrable;
(C) I have not had sufficient time to think about this yet.

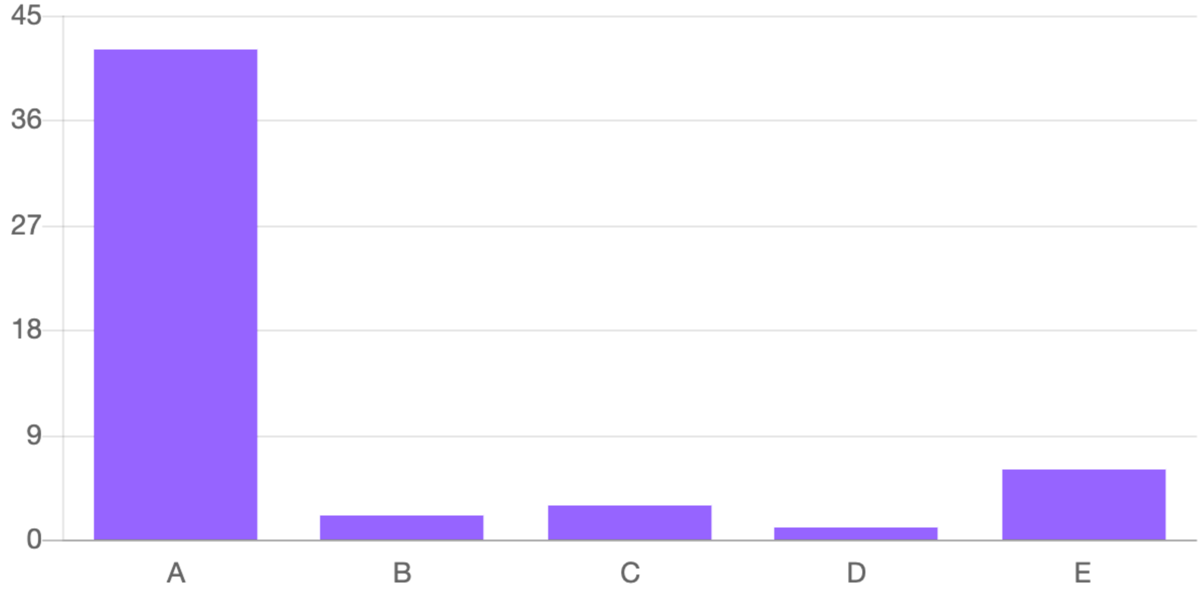


Question #2 Define $f : \mathbb{R} \to \mathbb{R}$ via $f(x) = x$ if $x \in \mathbb{Q}$ and $f(x) = 0$ if $x \notin \mathbb{Q}$.

(a) Let P be a partition of $[0, 1]$. Which of the following statements about $L(f, P)$ is true?

Note: The next two questions should really be labelled 2(b) and 2(c). childsmath does not allow subparts within a question. Apologies for any confusion.

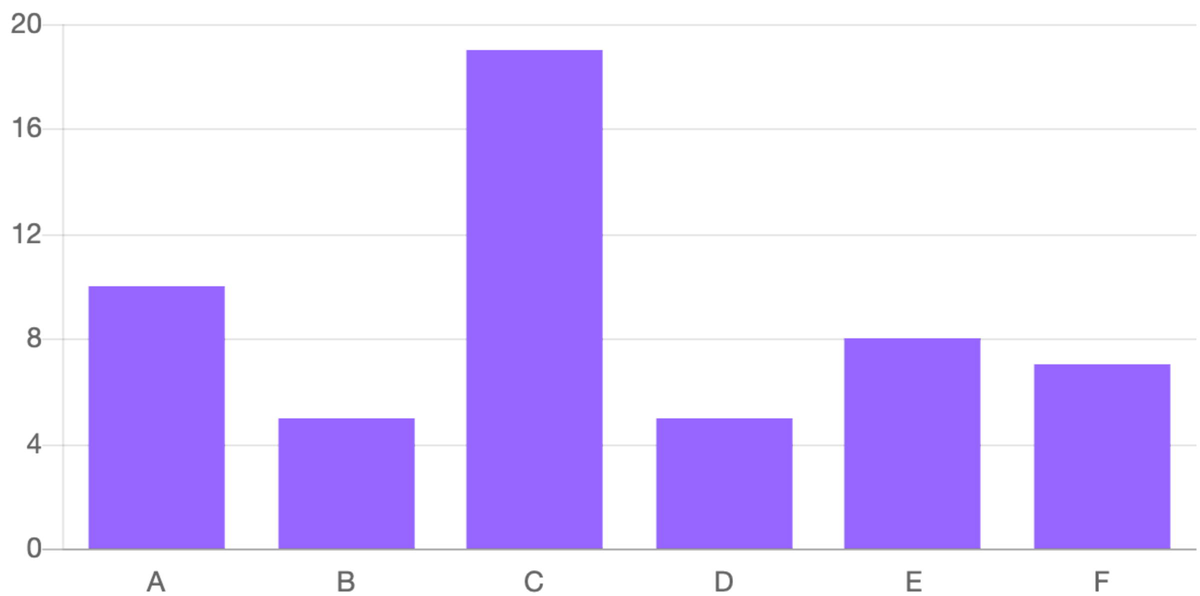
(A) $L(f, P) = 0$ for all P;
(B) $L(f, P) > 0$ for all P;
(C) $L(f, P) > 0$ for some P, but not all P;
(D) $L(f, P)$ can not be determined for any P;
(E) I have not had sufficient time to think about this yet.



Question #3 Recall from part (a) that $f(x) = x$ if $x \in \mathbb{Q}$ and $f(x) = 0$ if $x \notin \mathbb{Q}$.

(b) For convenience, denote $\inf\{U\} \equiv \inf\{U(f,P) : P \text{ a partition of } [0,1]\}$. Which of the following statements about $\inf\{U\}$ is correct?

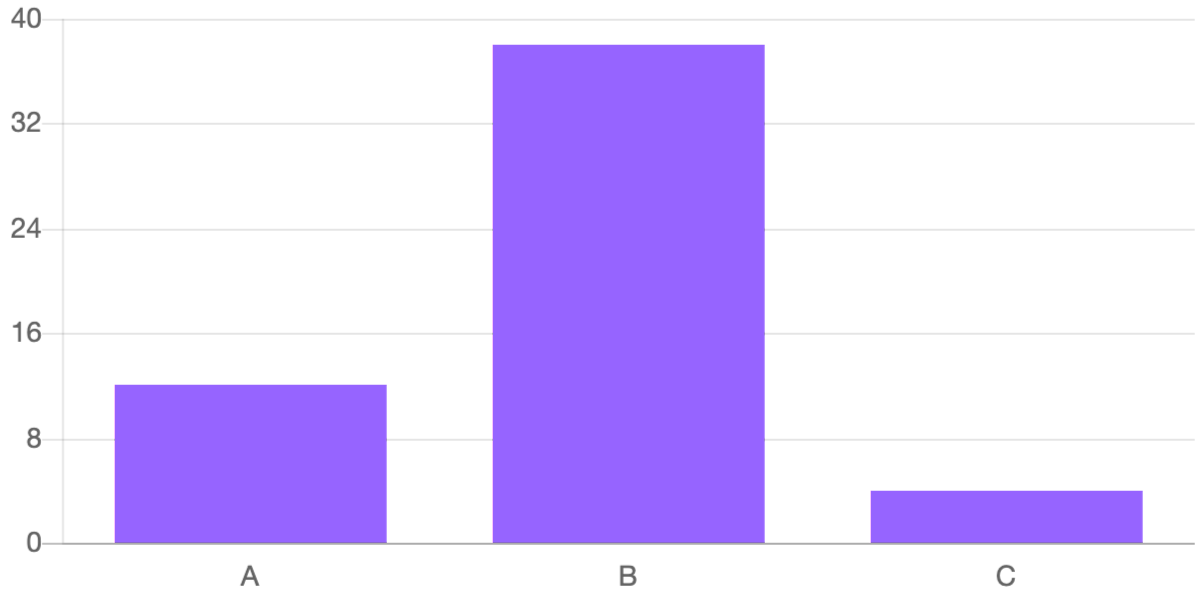
(A) $\inf\{U\} = 0$;
(B) $0 < \inf\{U\} < \frac{1}{2}$;
(C) $\inf\{U\} = \frac{1}{2}$;
(D) $\frac{1}{2} < \inf\{U\} < 1$;
(E) $\inf\{U\} = 1$;
(F) I have not had sufficient time to think about this yet.



Question #4 Recall from parts (a) and (b) that $f(x) = x$ if $x \in \mathbb{Q}$ and $f(x) = 0$ if $x \notin \mathbb{Q}$.

(c) Is f integrable on $[0,1]$?

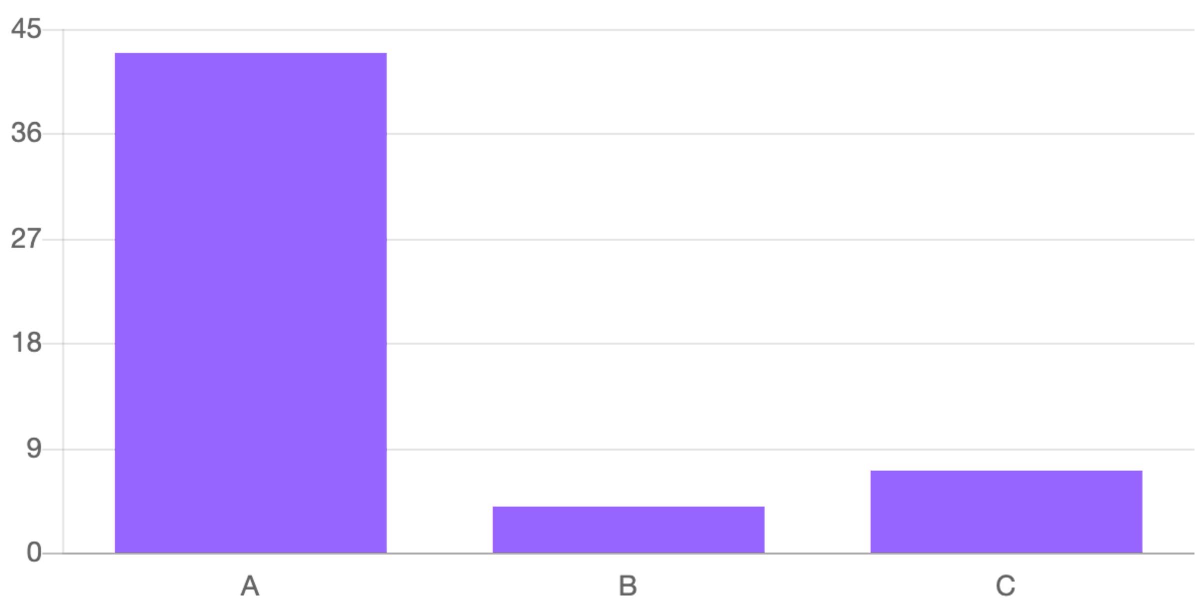
(A) Yes;
(B) No;
(C) I have not had sufficient time to think about this yet.



Question #5 A function is said to be *piecewise continuous* on an interval if the interval can be broken into a finite number of subintervals on which the function is continuous on each open subinterval (i.e., the subinterval without its endpoints) and has a finite limit at the endpoints of each subinterval.

Suppose $f : [a, b] \to \mathbb{R}$ is piecewise continuous. Prove whichever statement is true:

(A) f is integrable;
(B) f is not necessarily integrable;
(C) I have not had sufficient time to think about this yet.



Question #6 Recall that $\lceil x \rceil$ denotes the least integer that is greater than or equal to x. Let $f(x) = \lceil x \rceil$ for all $x \in \mathbb{R}$. Prove whichever of the following statements is true:

(A) $\int_0^2 f = 0;$

(B) $\int_0^2 f = 1;$

(C) $\int_0^2 f = 2;$

(D) $\int_0^2 f = 3;$

(E) $\int_0^2 f$ does not exist, i.e., f is not integrable;

(F) I have not had sufficient time to think about this yet.