# 13 Topology of $\mathbb{R}$ I

McMaster University

Mathematics and Statistics

$$\int_M d\omega = \int_{\partial M} \omega$$

# Mathematics 3A03
# Real Analysis I

Instructor: David Earn

Lecture 13
Topology of $\mathbb{R}$ I
Tuesday 1 October 2019

# THINKING ABOUT GRADUATE SCHOOL?

JOIN US TO FIND OUT MORE AT THE GRAD INFO SESSION!

**WHEN**: THURSDAY OCTOBER 3, 2019
**TIME**: 5:30PM – 7:00PM
**WHERE**: HH/305 AND THE MATH CAFÉ

Matheus Grasselli will give general advice on applying to grad school.

Shui Feng will talk about graduate programs particular to statistics.

Tom Hurd will talk about graduate opportunities in financial math including PhiMac.

Miroslav Lovric will give tips about applying to teachers' college.

**PIZZA** will be served! See you there!

# Announcements

- Assignment 3 is posted, but more problems will be added in a few days. **Due Tuesday 22 October 2019 at 2:25pm via crowdmark.**

# Topology of $\mathbb{R}$

# Intervals


*Open interval:*

$$(a, b) = \{x : a < x < b\}$$

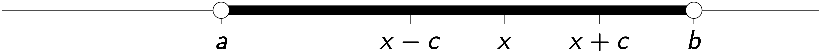
*Closed interval:*

$$[c, d] = \{x : c \leq x \leq d\}$$

*Half-open interval:*

$$(e, f] = \{x : e < x \leq f\}$$

# Interior point



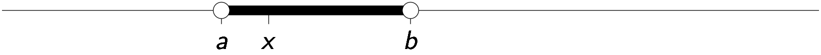

**Definition (Interior point)**

If $E \subseteq \mathbb{R}$ then $x$ is an ***interior point*** of $E$ if $x$ lies in an open interval that is contained in $E$, *i.e.*, $\exists c > 0$ such that $(x - c, x + c) \subset E$.

# Interior point examples

| **Set** $E$ | **Interior points?** |
|---|---|
| $(-1,1)$ | Every point |
| $[0,1]$ | Every point *except the endpoints* |
| $\mathbb{N}$ | $\nexists$ |
| $\mathbb{R}$ | Every point |
| $\mathbb{Q}$ | $\nexists$ |
| $(-1,1)\cup[0,1]$ | Every point *except 1* |
| $(-1,1)\setminus\{\frac{1}{2}\}$ | Every point |

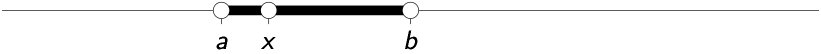
# Neighbourhood




**Definition (Neighbourhood)**

A ***neighbourhood*** of a point $x \in \mathbb{R}$ is an open interval containing $x$.

# Deleted neighbourhood

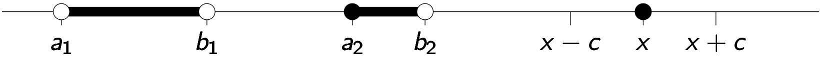



**Definition (Deleted neighbourhood)**

A ***deleted neighbourhood*** of a point $x \in \mathbb{R}$ is a set formed by removing $x$ from a neighbourhood of $x$.

$$(a, b) \setminus \{x\}$$

# Isolated point




$$E = (a_1, b_1) \cup [a_2, b_2) \cup \{x\}$$

## Definition (Isolated point)

If $x \in E \subseteq \mathbb{R}$ then $x$ is an ***isolated point*** of $E$ if there is a neighbourhood of $x$ for which the only point in $E$ is $x$ itself, *i.e.*, $\exists c > 0$ such that $(x - c, x + c) \cap E = \{x\}$.

# Poll

- Go to https://www.childsmath.ca/childsa/forms/main_login.php
- Click on Math 3A03
- Click on Take Class Poll
- Fill in poll **Lecture 13: Isolated points**
- Submit.

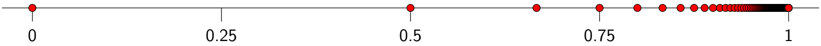
# Isolated point examples

| **Set** $E$ | **Isolated points?** |
|---|---|
| $(-1,1)$ | |
| $[0,1]$ | |
| $\mathbb{N}$ | |
| $\mathbb{R}$ | |
| $\mathbb{Q}$ | |
| $(-1,1) \cup [0,1]$ | |
| $(-1,1) \setminus \{\frac{1}{2}\}$ | |

# Isolated point examples

| **Set** $E$ | **Isolated points?** |
|---|---|
| $(-1,1)$ | $\nexists$ |
| $[0,1]$ | $\nexists$ |
| $\mathbb{N}$ | Every point |
| $\mathbb{R}$ | $\nexists$ |
| $\mathbb{Q}$ | $\nexists$ |
| $(-1,1)\cup[0,1]$ | $\nexists$ |
| $(-1,1)\setminus\{\frac{1}{2}\}$ | $\nexists$ |

# Accumulation point




$$E = \left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$$

## Definition (Accumulation Point or Limit Point)

If $E \subseteq \mathbb{R}$ then $x$ is an ***accumulation point*** or ***limit point*** of $E$ if every neighbourhood of $x$ contains infinitely many points of $E$,

$$\textit{i.e.,} \qquad \forall c > 0 \qquad (x-c, x+c) \cap (E \setminus \{x\}) \neq \varnothing \,.$$

*Notes:*

- It is possible but not necessary that $x \in E$.
- The shorthand condition is equivalent to saying that every deleted neighbourhood of $x$ contains at least one point of $E$.

# Poll

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- Submit.

# Accumulation point examples

| Set $E$ | Accumulation points? |
|---|---|
| $(-1,1)$ | |
| $[0,1]$ | |
| $\mathbb{N}$ | |
| $\mathbb{R}$ | |
| $\mathbb{Q}$ | |
| $(-1,1)\cup[0,1]$ | |
| $(-1,1)\setminus\{\frac{1}{2}\}$ | |
| $\left\{1-\frac{1}{n} : n\in\mathbb{N}\right\}$ | |

# Accumulation point examples

| Set $E$ | **Accumulation points?** |
|---|---|
| $(-1,1)$ | $[-1,1]$ |
| $[0,1]$ | $[0,1]$ |
| $\mathbb{N}$ | $\nexists$ |
| $\mathbb{R}$ | $\mathbb{R}$ |
| $\mathbb{Q}$ | $\mathbb{R}$ |
| $(-1,1)\cup[0,1]$ | $[-1,1]$ |
| $(-1,1)\setminus\{\frac{1}{2}\}$ | $[-1,1]$ |
| $\left\{1-\frac{1}{n} : n\in\mathbb{N}\right\}$ | $\{1\}$ |