Mathematics 3A03 Real Analysis I
Fall 2019 ASSIGNMENT 5

This assignment is **due** on **Thursday 21 November 2019 at 2:25pm**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. A function $f : \mathbb{R} \to \mathbb{R}$ is even if $f(-x) = f(x)$ for all x, and odd if $f(-x) = -f(x)$ for all x. Suppose f is differentiable. Prove, directly from the definition of the derivative, that (a) if f is even then f' is odd and (b) if f is odd then f' is even.

2. Establish that the hypotheses of Rolle's Theorem are necessary by constructing functions f that have the following properties, but for which it is not true that there exists $x \in (a, b)$ such that $f'(x) = 0$. In each case, state which hypothesis of Rolle's Theorem is not satified in your example.

   (a) f is continuous on $[a, b]$ and differentiable on (a, b);

   (b) f is continuous on $[a, b]$ and $f(a) = f(b)$;

   (c) f is differentiable on (a, b) and $f(a) = f(b)$.

3. Prove that if $a < b$ and f is integrable on the closed interval $[a, b]$ then f is necessarily integrable on any closed subinterval of $[a, b]$.

4. Define $f : \mathbb{R} \to \mathbb{R}$ via $f(x) = x$ if $x \in \mathbb{Q}$ and $f(x) = 0$ if $x \notin \mathbb{Q}$.

   (a) Let P be any partition of $[0, 1]$. Find $L(f, P)$.

   (b) Find $\inf\{U(f, P) : P \text{ a partition of } [0, 1]\}$.

   (c) Is f integrable on $[0, 1]$?

Version of November 10, 2019 @ 18:50