Mathematics 3A03 Real Analysis I
Fall 2019 ASSIGNMENT 4

This assignment is **due** on **Tuesday 12 November 2019 at 2:25pm**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. In each part of this problem, the function f is defined by the formula
$$f(x) = \sqrt{|x|}\,. \qquad (\heartsuit)$$
Pay close attention to the domain of the function in each part and consider the statement
$$\lim_{x\to 2} f(x) = \sqrt{2}\,. \qquad (\spadesuit)$$
Does statement ($\spadesuit$) make sense for the given domain? If not, why not? If statement ($\spadesuit$) does make sense, then either prove or disprove it directly from the ε-δ definition of a limit.
   (a) $f : \mathbb{R} \to \mathbb{R}$.
   (b) $f : \mathbb{Q} \to \mathbb{R}$.
   (c) $f : \mathbb{Z} \to \mathbb{R}$.

2. The ***floor*** function is defined for all $x \in \mathbb{R}$ by $\lfloor x \rfloor$ = the greatest integer less than or equal to x, *i.e.,* the greatest $n \in \mathbb{Z}$ such that $n \le x$. Determine the points of continuity of the following functions:
   (a) $f(x) = \lfloor x \rfloor$;
   (b) $f(x) = x \lfloor x \rfloor$;
   (c) $f(x) = \lfloor 1/x \rfloor$, $x \neq 0$.

3. Show that the function $f(x) = 1/x^2$ is (a) uniformly continuous on $[1, \infty)$, but (b) not uniformly continuous on $(0, \infty)$.

4. Prove that a continuous function maps closed intervals to closed intervals. Thus, if $a < b$ and $f : [a, b] \to \mathbb{R}$ is continuous then $f([a, b])$ (the range of f) is a closed interval. *Hint:* Intermediate Value Theorem.

   Note: This is a special case of the more general theorem mentioned in class that a continuous function maps compact sets to compact sets.

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