Mathematics 3A03 Real Analysis I
Fall 2019 ASSIGNMENT 2

This assignment is **due** on **Tuesday 1 October 2019 at 2:25pm**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Use the formal definition of a limit of a sequence to prove that

   (a) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$;

   (b) $\lim_{n\to\infty} \frac{n^n - 1}{n^n + 1} = 1$.

2. Use the formal definition to prove that the following sequences $\{a_n\}$ *diverge* as $n \to \infty$.

   (a) $a_n = \sqrt{n}$;

   (b) $a_n = n^{1/k}$ (for fixed $k \in \mathbb{N}$).

3. (a) Prove that $\lim_{n\to\infty} a_n = 0$ if and only if $\lim_{n\to\infty} |a_n| = 0$.

   (b) Give an example to show that convergence of $\{|a_n|\}$ need not imply convergence of $\{a_n\}$.

4. Suppose $\lim_{n\to\infty} a_n = a$ and $a > 0$. Prove that

   (a) $\exists N \in \mathbb{N}$ such that $a_n > 0$, $\forall n \geq N$;

   (b) $\exists N' \in \mathbb{N}$ such that $\frac{1}{2}a < a_n < 2a$, $\forall n \geq N'$.