Mathematics 3A03 Real Analysis I
Fall 2019 ASSIGNMENT 1

This assignment is **due** on **Tuesday 17 September 2019 at 2:25pm**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Prove or disprove: $\sqrt{3/2}$ is irrational.

2. What is wrong with the following "proof"? Let $x = y$. Then

$$\begin{aligned} x^2 &= xy, \\ x^2 - y^2 &= xy - y^2, \\ (x+y)(x-y) &= y(x-y), \\ x + y &= y, \\ 2y &= y, \\ 2 &= 1. \end{aligned}$$

3. Prove the following:

   (a) $|x - y| \leq |x| + |y|$. (Give a very short proof.)

   (b) $|x| - |y| \leq |x - y|$. (A very short proof is possible, if you write things in the right way.)

   (c) $|(|x| - |y|)| \leq |x - y|$. (Why does this follow immediately from (3b)?)

   (d) $|x + y + z| \leq |x| + |y| + |z|$. (Indicate when equality holds, and prove your statement.)

4. Prove by induction that if $x > -1$ then $(1 + x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$.

5. For each of the following sets, find the greatest lower bound (inf), least upper bound (sup), minimum (min) and maximum (max), if they exist. If any of these do not exist, then indicate accordingly. Justify your assertions.

   (a) $(1, 2) \cup (2, 3] \cup (-3, -2] \cup (-2, -1)$.

   (b) $\{p^q : p, q \text{ prime}\}$.

   (c) $\{x \in \mathbb{R} : x < 1/x\}$.

6. Suppose A and B are bounded subsets of $\mathbb{R}$. Prove that $A \cup B$ is bounded and $\sup(A \cup B) = \sup\{\sup A, \sup B\}$.