Mathematics 3A03 Real Analysis I

http://www.math.mcmaster.ca/earn/3A03

2019 ASSIGNMENT 4

This assignment is **due** on **Friday 8 March 2019 at 1:25pm**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Give an example of a sequence of closed sets F_1, F_2, F_3, $\ldots$, whose union is neither open nor closed. Can this be achieved with a sequence that contains only finitely many distinct sets?

2. Suppose that $E \subseteq \mathbb{R}$, $K \subseteq \mathbb{R}$, E is closed and K is compact. Show that $E \cap K$ is compact, by proving directly that $E \cap K$ satisfies each of the following equivalent properties:

   (a) closed and bounded;

   (b) Bolzano-Weierstrass property;

   (c) Heine-Borel property.

3. For which of the following functions f is there a continuous function g with domain $\mathbb{R}$ such that $g(x) = f(x)$ for all x in the domain of f?

   (i) $f(x) = \dfrac{x^2 - 4}{x - 2}$,

   (ii) $f(x) = \dfrac{|x|}{x}$,

   (iii) $f(x) = 0$, x irrational.

4. Prove that if f is continuous at a, then for any $\varepsilon > 0$ there is a $\delta > 0$ such that whenever $|x - a| < \delta$ and $|y - a| < \delta$, we have $|f(x) - f(y)| < \varepsilon$.

5. Suppose $a, b \in \mathbb{R}$ and $a < b$. Prove directly from the definition that $f(x) = x^2$ is uniformly continuous on the closed interval $[a, b]$. Is f uniformly continuous on the open interval (a, b)?

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