Mathematics 3A03 Real Analysis I

http://www.math.mcmaster.ca/earn/3A03

2019 ASSIGNMENT 3

This assignment is **due** on **Friday 15 Feb 2019 at 1:25pm**.
PLEASE NOTE that you must **submit online** via crowdmark.
You should have received an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Let $\{x_n\}$ be a bounded sequence and let $x = \sup\{x_n : n \in \mathbb{N}\}$. Suppose that, moreover, $x_n < x$ for all n. Prove that there is a subsequence of $\{x_n\}$ that converges to x.
2. Show directly that the sequence $s_n = \frac{1}{n}$ is a Cauchy sequence.
3. Show directly that if $\{s_n\}$ is a Cauchy sequence then so too is $\{|s_n|\}$. From this conclude that $\{|s_n|\}$ converges whenever $\{s_n\}$ converges.
4. Define a sequence $\{a_n\}$ recursively by setting $a_1 = 1$ and $a_n = \sqrt{1 + a_{n-1}}$. Prove that $\{a_n\}$ converges and find its limit. *Hint:* First show by induction that a_n is bounded.
5. Suppose $A \subset B \subset \mathbb{R}$. Prove that if B is countable then A is countable.
6. Suppose $A \subset \mathbb{R}$ and $B = \{b : b = a \text{ or } b = a^2 \text{ for some } a \in A\}$. Prove that if A is countable then B is countable.
7. Show (a) that no interior point of a set can be a boundary point, (b) that it is possible for an accumulation point to be a boundary point, and (c) that every isolated point must be a boundary point.
8. Express the closed interval $[0, 1]$ as an intersection of a sequence of open sets. Can it also be expressed as a union of a sequence of open sets?