Mathematics 3A01 Real Analysis I
2017 ASSIGNMENT 2

This assignment is **due in the appropriate locker** on **Friday 29 Sep 2017 at 4:25pm**.

1. Use the principle of mathematical induction to prove that for any $n \in \mathbb{N}$,

   (a) $\displaystyle\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$;

   (b) if the integers $x_1, \ldots, x_n$ are odd then their product $x_1 x_2 \cdots x_n$ is odd.
   Hint: Start with $n = 2$.

2. Use the formal definition of a limit of a sequence to prove that

   (a) $\displaystyle\lim_{n\to\infty} \frac{(-1)^n}{n^3} = 0$;

   (b) $\displaystyle\lim_{n\to\infty} \frac{n^2 - 1}{n^2 + 1} = 1$.

3. Use the formal definition to prove that the following sequences $\{s_n\}$ *diverge* as $n \to \infty$.

   (a) $s_n = (-r)^n$ (for any $r \geq 1$) ;

   (b) $s_n = \dfrac{n!}{2^n}$.

4. Suppose $s_n \to 0$ as $n \to \infty$ and that $s_n > 0$ for at least one $n \in \mathbb{N}$. Prove that the set $\{s_n\}$ has a maximum value.

5. Prove that if $\displaystyle\lim_{n\to\infty} s_n = L$ then $\displaystyle\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} s_k = L$.