Mathematics 3A03 Real Analysis I
2017 ASSIGNMENT 1

This assignment is **due in the appropriate locker** on **Friday 15 Sep 2017 at 4:25pm**.

1. Prove that $-2\sqrt{2}+3$ is irrational.

2. Define $\mathbb{Z}_4$ to consist of the set $\{0,1,2,3\}$, together with addition $+$ and multiplication $\cdot$ defined by the following two tables.

$+$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\cdot$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

   Is $\mathbb{Z}_4$ a field? Justify your answer.

3. For each of the following sets, find the greatest lower bound (inf), least upper bound (sup), minimum (min) and maximum (max), if they exist. If any of these do not exist, then indicate accordingly. Justify your assertions.

   (a) $(-\infty, 2]$.

   (b) $\{x : x \in \mathbb{R} \text{ and } |x| < 3\}$.

   (c) $\{\frac{n+1}{n} : n \in \mathbb{N}\}$.

4. Suppose $S, T \subseteq \mathbb{R}$ are bounded, nonempty sets with $S \subseteq T$. Find relations between $\sup S, \sup T, \inf S$ and $\inf T$. Justify your assertions.

5. Let $x, y \in \mathbb{R}$. Prove that $x = y$ if and only if $|x-y| < \epsilon$ for every $\epsilon > 0$.

6. Let $x_1, x_2 \in \mathbb{R}$ be real numbers with $x_1 < x_2$. Show that if $y \in (x_1, x_2)$, then there are rational numbers $r_1, r_2 \in \mathbb{Q}$ with $y \in (r_1, r_2)$ and $(r_1, r_2) \subseteq (x_1, x_2)$.