Mathematics 3A01 Real Analysis I
2016 ASSIGNMENT 1

This assignment is **due in the appropriate locker** on **Friday 16 Sep 2016 at 4:25pm**.

1. Prove that $\sqrt{3}$ is irrational.

2. The field of integers modulo 2 ($\mathbb{Z}_2$) can be defined by interpretting "number" to mean either 0 or 1, and + and $\cdot$ to be the operations specified by the following two tables.

+	0	1
0	0	1
1	1	0

$\cdot$	0	1
0	0	0
1	0	1

   Prove that all the field axioms hold for $\mathbb{Z}_2$, even though $1 + 1 = 0$.

3. For each of the following sets, find the greatest lower bound (inf), least upper bound (sup), minimum (min) and maximum (max), if they exist, or indicate non-existence ($\nexists$). Justify your assertions.

   (a) $(-2, -1)$.
   (b) $\{\frac{1}{n} : n \in \mathbb{N}\}$.
   (c) $\{\frac{1}{n} : n \in \mathbb{Z} \text{ and } n \neq 0\}$.
   (d) $\{\frac{1}{n} + (-1)^n : n \in \mathbb{N}\}$.

4. Prove that if

$$|x - x_0| < \frac{\varepsilon}{2} \qquad \text{and} \qquad |y - y_0| < \frac{\varepsilon}{2},$$

   then

$$|(x + y) - (x_0 + y_0)| < \varepsilon,$$
$$\text{and } |(x - y) - (x_0 - y_0)| < \varepsilon.$$

5. Suppose q is a rational number such that

$$1 < q \leq 3 - \sqrt{2}. \tag{*}$$

   Write $q = m/n$, where $m, n \in \mathbb{N}$ and $\gcd(m, n) = 1$. Prove that $\sqrt{2}$ is a positive distance from q. Specifically, show that

$$\left|\sqrt{2} - \frac{m}{n}\right| \geq \frac{1}{3n^2}. \tag{**}$$

   Hint: First prove that if $\gcd(m, n) = 1$ then $2n^2$ and m^2 are distinct integers; then "rationalize the numerator" in the LHS of (**).